

Continuous-Aperture Array (CAPA)- Based Wireless Communications: When Hertz Meets Leibniz



Prof. Yuanwei Liu

Professor, IEEE Fellow, AAIA Fellow

Web of Science Highly Cited Researcher,

IEEE Communication Society Distinguished Lecturer,

IEEE Vehicular Technology Distinguished Lecturer

e-mail: yuanwei@hku.hk

website: <https://www.eee.hku.hk/~yuanwei/>



香 港 大 學

THE UNIVERSITY OF HONG KONG

Outline

□ CAPA Basis

- Overview of CAPA (**What**)
- Motivation of Employing CAPA (**Why**)
- Application of CAPA (**How**)

□ Case Study

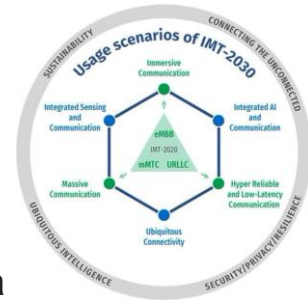
- Case I: Information-Theoretic Limits for CAPA
- Case II: Performance Analysis for CAPA
- Case III: Beamforming Design for CAPA
- Research Challenges and Opportunities



Overview of CAPA towards 6G

❑ A Concise Timeline for Multiple-Antenna Technologies

- Providing *array* gain, *diversity* gain, *multiplexing* gain
- *Reliability*, *effectiveness*, and *security* improvements



Hertz

Karl Braun

Beverage

Bruce Bolt et al.

Jack Winter

Foschini et al.

T. Marzetta

1887

1905

1926

1960s

1984

1990s

2010s

2020s

Dipole antenna

Phased array, beamforming

Space diversity

Digital antenna array

Optimal combining

MIMO BLAST

Massive MIMO

ELAA & RIS & Flexible & Fluid & Movable & Holographic MIMO

❑ Evolution Trend

Larger (in aperture size), **Denser** (in antenna deployment), **Higher** (in frequency), and **more Flexible** (in aperture shape)

[1] Y. Liu, C. Ouyang, Z. Wang, J. Xu, X. Mu, and Z. Ding, "CAPA: Continuous-aperture arrays for revolutionizing 6G wireless communications," *IEEE Wireless Commun.*, vol. 32, no. 4, pp. 38-45, Aug. 2025. <https://arxiv.org/abs/2412.00894>



香港大學

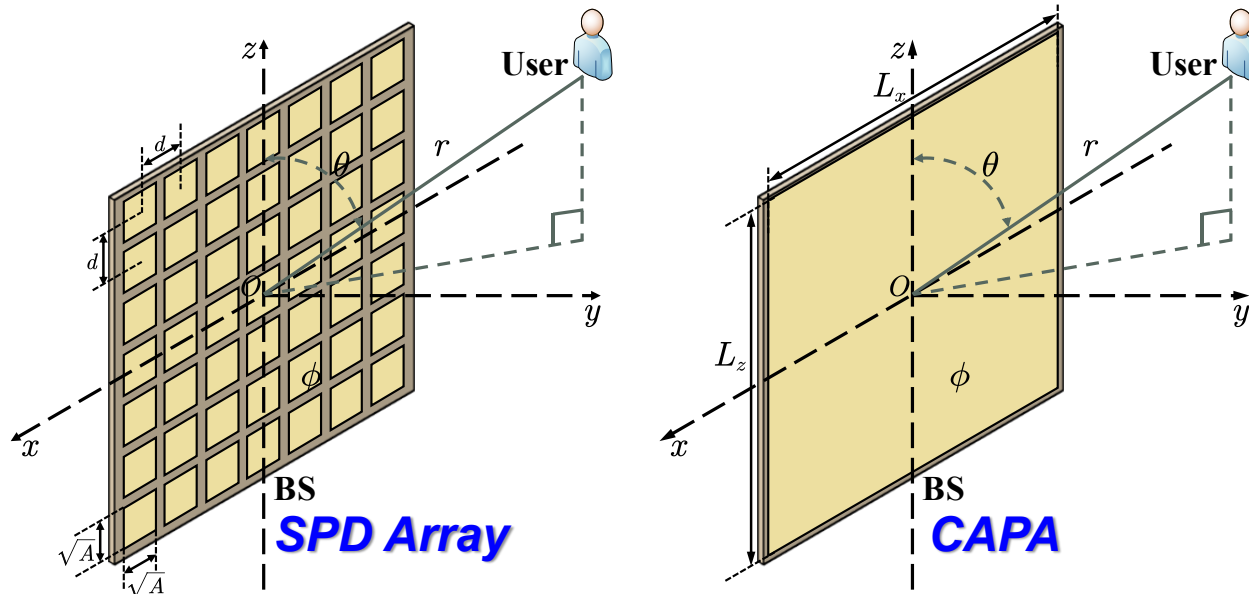
THE UNIVERSITY OF HONG KONG



Overview: What is CAPA?

❑ Continuous-Aperture Array: Ultimate Version of MIMO

- An array that forms a *spatially continuous electromagnetic (EM) aperture*
- A shift from the conventional spatially discrete (SPD) array
- A CAPA = **One single antenna** with large aperture size and **continuous current source**
- Controlling the EM wave via controlling the *source current* or *excited electric field*

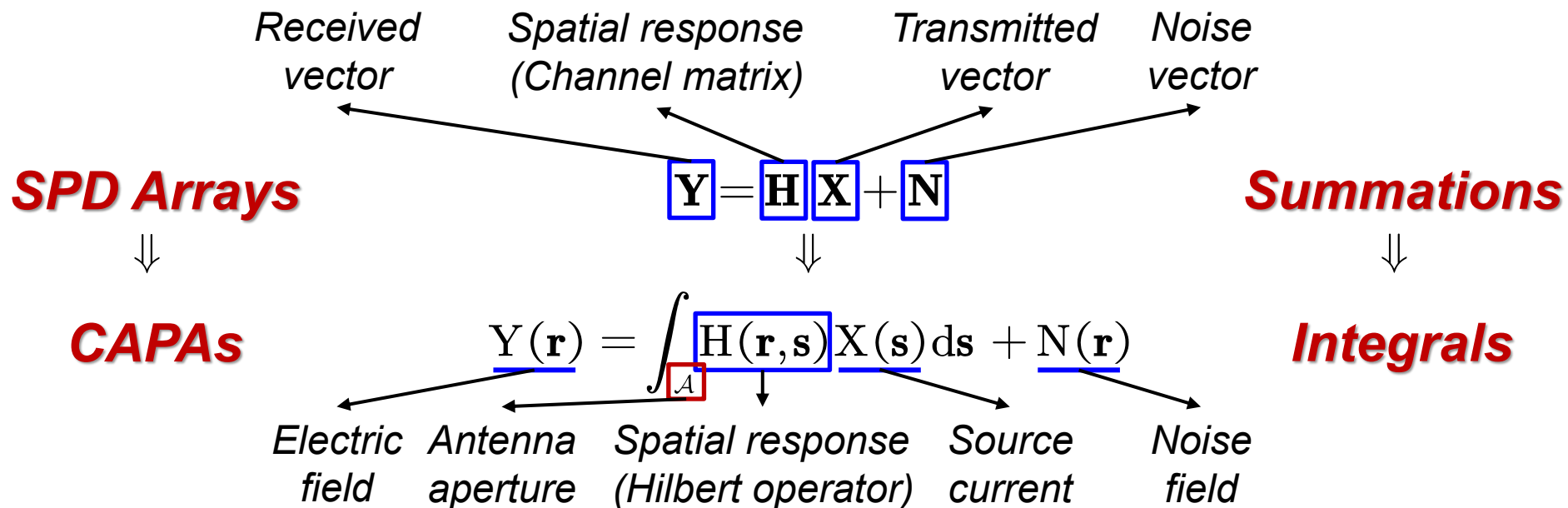


[1] Y. Liu, C. Ouyang, Z. Wang, J. Xu, X. Mu, and Z. Ding, "CAPA: Continuous-aperture arrays for revolutionizing 6G wireless communications," *IEEE Wireless Commun.*, vol. 32, no. 4, pp. 38-45, Aug. 2025. <https://arxiv.org/abs/2412.00894>

What will be different?

❑ Different Mathematical Model

- From *Matrix-based Discrete Model* to *Hilbert Operator-based Continuous Model*



- Challenges for *Hilbert Operator*: no explicit form for inverse/decomposition

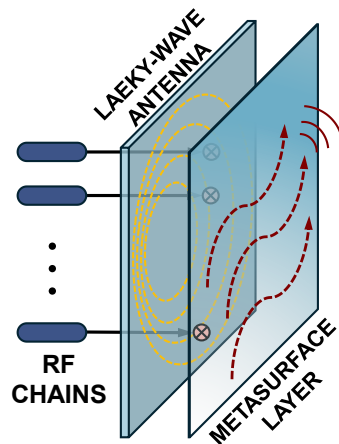
[1] Y. Liu, C. Ouyang, Z. Wang, J. Xu, X. Mu, and Z. Ding, "CAPA: Continuous-aperture arrays for revolutionizing 6G wireless communications," *IEEE Wireless Commun.*, vol. 32, no. 4, pp. 38-45, Aug. 2025. <https://arxiv.org/abs/2412.00894>



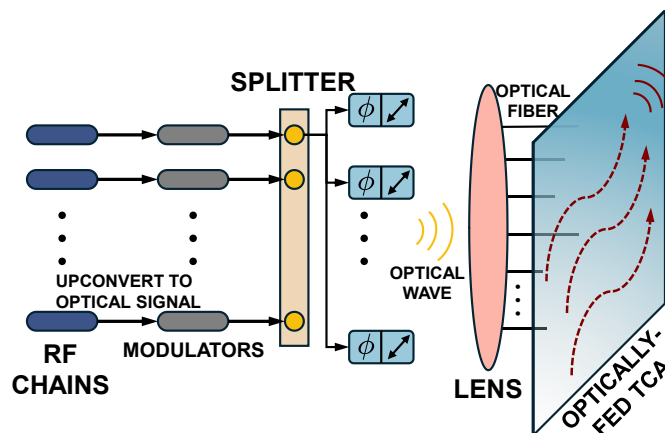
What will be different?

□ Different Hardware Implementation

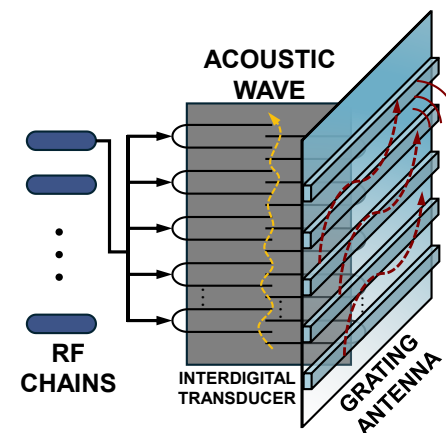
- **Independent phase/amplitude control** of the EM wave: (i) electrically, (ii) optically, (iii) acoustically
- **Spatially continuous control** across the entire aperture: (i) tightly-coupled arrays, (ii) leaky-wave antennas



Electrically driven



Optically driven

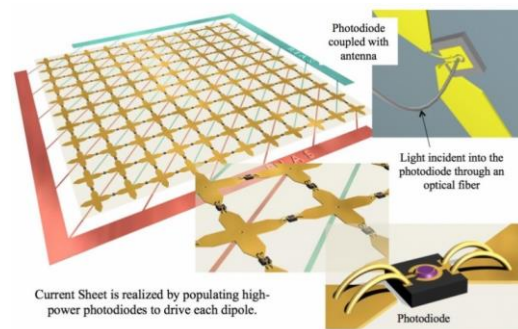


Acoustically driven

- [1] Y. Liu, C. Ouyang, Z. Wang, J. Xu, X. Mu, and Z. Ding, "CAPA: Continuous-aperture arrays for revolutionizing 6G wireless communications," *IEEE Wireless Commun.*, vol. 32, no. 4, pp. 38-45, Aug. 2025. <https://arxiv.org/abs/2412.00894>
- [2] M. Rezvani, R. Adve, A. Bin Sediq, and A. El-Keyi, "Energy efficient wireless communications by harnessing Huygens' metasurfaces," *IEEE J. Sel. Areas Inf. Theory*, vol. 6, pp. 85-99, May 2025.
- [3] D. W. Prather, S. Shi, G. J. Schneider, P. Yao, C. Schuetz, J. Murakowski, J. C. Deroba, F. Wang, M. R. Konkol, and D. D. Ross, "Optically upconverted, spatially coherent phased-array-antenna feed networks for beamspace MIMO in 5G cellular communications," *IEEE Trans. Antennas Propag.*, vol. 65, no. 12, pp. 6432-6443, Dec. 2017.
- [4] Y. Yuan, C. Jiang, and J. Zhu, "Interdigital waveguide grating antenna array for an optical phased array," *Appl. Opt.*, vol. 63, no. 28, pp. 7370-7377, Oct. 2024.

What will be different?

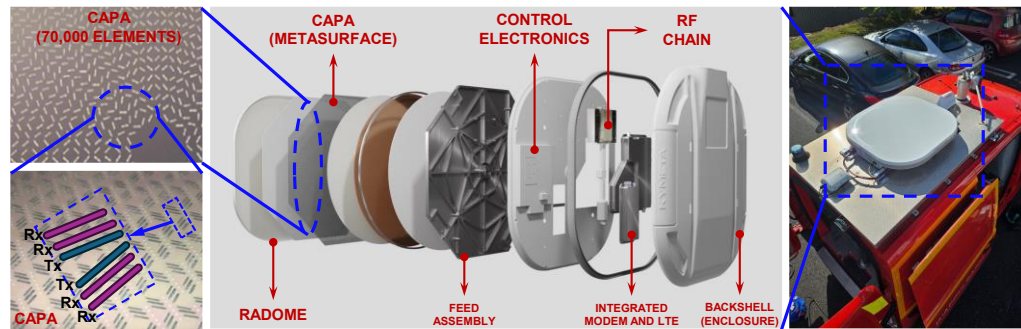
❑ Different Hardware Implementation



Optically driven tightly coupled arrays
University of Delaware



Metasurface
Pivotal Commware
Backed by **Bill Gates**



Metasurface; Kymeta Corporation
70,000 radiating elements, each equipped with a tunable capacitor, into a circular aperture with an **82cm** diameter

Realizing a (near-)continuous EM aperture is *physically achievable* in practice. This makes CAPA a promising research topic in 6G due to its performance gains.

Whether approximating or realizing a physically continuous CAPA, the underlying signal model is commonly characterized using a *Hilbert operator*.

- [1] **Y. Liu**, C. Ouyang, Z. Wang, J. Xu, X. Mu, and Z. Ding, "CAPA: Continuous-aperture arrays for revolutionizing 6G wireless communications," *IEEE Wireless Commun.*, vol. 32, no. 4, pp. 38-45, Aug. 2025. <https://arxiv.org/abs/2412.00894>
- [2] D. W. Prather, S. Shi, G. J. Schneider, P. Yao, C. Schuetz, J. Murakowski, J. C. Deroba, F. Wang, M. R. Konkol, and D. D. Ross, "Optically upconverted, spatially coherent phased-array-antenna feed networks for beamspace MIMO in 5G cellular communications," *IEEE Trans. Antennas Propag.*, vol. 65, no. 12, pp. 6432-6443, Dec. 2017.
- [3] E. J. Black, "Holographic beam forming and MIMO," Pivotal Commware, Tech. Rep., 2017; <https://pivotalcommware.com/wpcontent/uploads/2019/10/HBF-vs-APA-White-Paper-2019.pdf>.
- [4] M. Sazegar, I. Nassar, C. Eylander, A. Momeni, and R. Stevenson, "Full duplex SATCOM ESA with switchable polarization and wide tunable bandwidth using a tripleband metasurface aperture," in 2022 IEEE Int. Symp. Antennas Propag. and USNC-URSI Radio Sci. Meeting (APS/URSI), 2022, pp. 639-640.



香港大學

THE UNIVERSITY OF HONG KONG

Mingle with Two Pioneers~ :-)

❑ CAPA: When Hertz Meets Leibniz



@Hertz
Inventor of the **Antenna**



**CAPA-based
Communications**



@Leibniz
Inventor of the **Integral**

❑ CAPA-Based Wireless Communications

*Establish a framework and associated methodology
for CAPA-based wireless communications towards 6G*



Why do we need CAPA?

❑ From an Antenna Manufacturing Perspective

- Achieving an ideal CAPA has been a *long-standing goal* in antenna design

❑ From a Wireless Transmission Perspective

- CAPAs can achieve *greater array, multiplexing, and diversity gains* than SPDAs

❑ Beyond Manufacturing and Transmission

- CAPA represents *not only an advanced array technology* but also *a conceptual framework* that offers a new approach for designing and analyzing future arrays

- ❑ **Beamforming Optimization:** Optimizing a continuous current distribution—essentially a function—can be computationally more efficient than optimizing high-dimensional matrices,
- ❑ **Performance Analysis:** Integrals, as opposed to discrete summations, are often more analytically convenient and can provide deeper insights
- ❑ **Storage Efficiency:** Storing a continuous function can be more memory-efficient compared to storing a large matrix, i.e. Q-Learning to DQN
- ❑ **Applications:** Optimizing a dense array from the CAPA perspective allows for a continuum approach



Motivation of Employing CAPA

❑ Comparison Between CAPA and SPD Arrays

	CAPA	SPD Array
<i>Implementation</i>	Meta-surface	Patch elements
<i>Model</i>	Operator-based (continuous)	Matrix-based (discrete)
<i>Transmission</i>	Currents-based	Beamformer-based
<i>Number of DoFs</i>	Upper Bound	Achievable value
<i>Array Gain</i>	Upper Bound	Achievable value
<i>Diversity Gain</i>	Upper Bound	Achievable value
<i>Multiplexing Gain</i>	Upper Bound	Achievable value

- CAPA has many superior advantages over SPD arrays
- The optimization and implementation of CAPA is more *challenging*

[1] Y. Liu, C. Ouyang, Z. Wang, J. Xu, X. Mu, and Z. Ding, "CAPA: Continuous-aperture arrays for revolutionizing 6G wireless communications," *IEEE Wireless Commun.*, vol. 32, no. 4, pp. 38-45, Aug. 2025. <https://arxiv.org/abs/2412.00894>



Outline

□ CAPA Basis

- Overview of CAPA (**What**)

- Motivation of Employing CAPA (**Why**)

- Application of CAPA (**How**)

□ Case Study

- Case I: Information-Theoretic Limits for CAPA

- Case II: Performance Analysis for CAPA

- Case III: Beamforming Design for CAPA

□ Research Challenges and Opportunities



Information-Theoretic Limits for CAPA

❑ CAPA-based Multiuser Channel

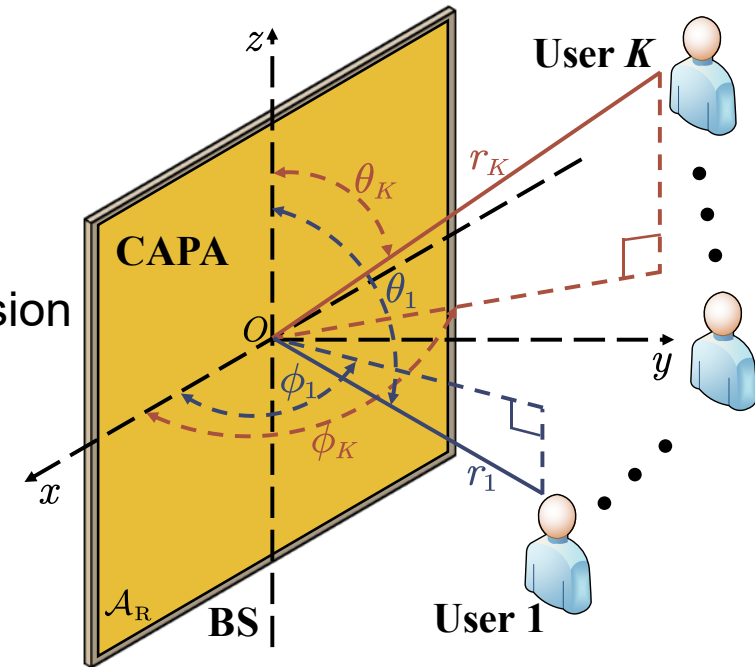
- User: a hypothetical **isotropic antenna**; Base Station (BS): a **CAPA**
- SPD Array: **matrix-based** UL/DL transmission

$$\mathbf{y} = \sum_{k=1}^K \mathbf{h}_k s_k + \mathbf{n} \quad y_k = \mathbf{h}_k^H \sum_{j=1}^K \mathbf{w}_j s_j + n_k$$

- CAPA: Hilbert **operator-based** UL/DL transmission
- From **matrix manipulations** to **integrals**

❑ Multiuser Channel Capacity

- Channel Capacity: *sum-rate, capacity region*
- Capacity-Achieving Strategies: *precoding/detection*



[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>

Information-Theoretic Limits for CAPA: Uplink

Multiuser Case

$$\underbrace{Y_{\text{ul}}(\mathbf{r})}_{\text{Received signal}} = \sum_{k=1}^K \underbrace{s_{\text{ul},k}}_{\text{Coded data symbol}} \int \underbrace{\mathcal{A}_k}_{\text{User } k\text{'s aperture}} \underbrace{H(\mathbf{r}, \mathbf{s}) J_{\text{ul},k}(\mathbf{s})}_{\text{User } k\text{'s spatial response}} \underbrace{ds}_{\text{User } k\text{'s source current}} + \underbrace{N_{\text{ul}}(\mathbf{r})}_{\text{Noise random field}} \Leftarrow \mathbf{y} = \sum_{k=1}^K \mathbf{h}_k s_{\text{ul},k} + \mathbf{n}$$

Simplification: Point-Like User (Integral $\rightarrow$ Multiplication)

$$Y_{\text{ul}}(\mathbf{r}) \approx \sum_{k=1}^K s_{\text{ul},k} |\mathcal{A}_k| H(\mathbf{r}, \mathbf{s}_{\text{u},k}) J_{\text{ul},k}(\mathbf{s}_{\text{u},k}) + N_{\text{ul}}(\mathbf{r}) \Leftarrow \mathbf{y} = \sum_{k=1}^K \mathbf{h}_k s_{\text{ul},k} + \mathbf{n}$$

What Shall We do? Continuous Signal to Messages

$$Y_{\text{ul}}(\mathbf{r}) \xrightarrow{\text{Detector}} \hat{s}_{\text{ul},k} \big|_{k=1, \dots, K} \xrightarrow{\text{Decoder}} \text{Messages in } s_{\text{ul},k} \big|_{k=1, \dots, K}$$

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>

Information-Theoretic Limits for CAPA: Uplink

❑ Multiuser Case Study: Two-User Channel

$$Y_{\text{ul}}(\mathbf{r}) \approx s_{\text{ul},1} |\mathcal{A}_1| H(\mathbf{r}, \mathbf{s}_{\text{u},1}) J_{\text{ul},1}(\mathbf{s}_{\text{u},1}) + s_{\text{ul},2} |\mathcal{A}_2| H(\mathbf{r}, \mathbf{s}_{\text{u},2}) J_{\text{ul},2}(\mathbf{s}_{\text{u},2}) + N_{\text{ul}}(\mathbf{r})$$

❑ Capacity-Achieving Detection/Decoding: SIC-ML

- **SIC + ML Decoding**: the message of one user is first decoded by *treating* the other user's message as *interference*; Once decoded, this message is *removed*, and the other message is then decoded *without* inter-user *interference* (IUI).
- SIC decoding order $1 \rightarrow 2$, SIC decoding order $2 \rightarrow 1$

❑ SIC-based Decoding for CAPA

$$\begin{array}{ccccccc} Y_{\text{ul}}(\mathbf{r}) & \mapsto & \int_{\mathcal{A}} (\cdot) d\mathbf{r} & \mapsto & s_{\text{ul},2} & \mapsto & Z(\mathbf{r}) \mapsto \int_{\mathcal{A}} (\cdot) d\mathbf{r} \mapsto s_{\text{ul},1} \\ \text{?} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \text{MRC} \\ & & & & \downarrow & & \\ & & & & s_{\text{ul},1} |\mathcal{A}_1| H(\mathbf{r}, \mathbf{s}_{\text{u},1}) J_{\text{ul},1}(\mathbf{s}_{\text{u},1}) + N_{\text{ul}}(\mathbf{r}) & \text{Single-user case} \end{array}$$

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Information-Theoretic Limits for CAPA: Uplink

❑ SIC-based Decoding: Whitening the Interference

- The simplest case: SU case with white noise (zero-mean, Dirac delta correlation)
- *Our case: SU case with non-white noise plus interference $Z(r)$*
- *Our strategy: whiten the interference to make it zero mean and delta correlated*

❑ How to Design a Whitening Filter $W(r', r)$

- Filtering: $\int_A W(r', r) Z(r) dr = Z_W(r')$
- **Whitening:** the autocorrelation of $Z_W(r')$ is proportional to the *Dirac delta function*
- **Invertibility:** For any $f(r)$, it has $\int_A W_1(r, r') \int_A W(r', r) f(r) dr dr' = f(r)$
- *Why invertible: **information lossless** or **capacity lossless***

$$\underline{W_Z(\mathbf{r}', \mathbf{r}) = \delta(\mathbf{r}' - \mathbf{r}) + \mu_1 H_1(\mathbf{r}', \mathbf{s}_{u,1}) H_1^*(\mathbf{r}, \mathbf{s}_{u,1})} \Leftarrow \underline{(\mathbf{I} + \mu_1 \mathbf{h}_1 \mathbf{h}_1^H)^{-1/2}}$$

MMSE Filtering for CAPA

MMSE Filtering for SPD

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Information-Theoretic Limits for CAPA: Uplink

❑ SIC Decoding for Uplink CAPA Communications

- *Whiten the interference-plus-noise term $\int_{\mathcal{A}} W(r', r) Z(r) dr = Z_w(r')$*
- *Use the MRC detector and ML decoder to recover $s_{ul,2}$ from $\int_{\mathcal{A}} W(r', r) Y_{ul}(r) dr$*
- *Employ SIC to subtract $H(r, s_{u,2})J_{ul,2}(s_{u,2})A_{u,2} s_{ul,2}$ from $Y_{ul}(r)$*
- *Use the MRC detector and ML decoder to recover $s_{ul,1}$ from $Z(r)$*

❑ Per-User Rate and Sum-Rate Capacity

$$\log_2(1 + \bar{\gamma}_1 \|\mathbf{h}_1\|^2) \Rightarrow \log_2\left(1 + \bar{\gamma}_1 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,1})|^2 d\mathbf{r}\right)$$
$$\log_2(1 + \bar{\gamma}_2 \mathbf{h}_2^H (\mathbf{I} + \bar{\gamma} \mathbf{h}_1 \mathbf{h}_1^H)^{-1} \mathbf{h}_2) \Rightarrow \log_2\left(1 + \bar{\gamma}_2 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,2})|^2 d\mathbf{r} \left(1 - \frac{\bar{\gamma}_1 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,1})|^2 d\mathbf{r} |\rho|^2}{1 + \bar{\gamma}_1 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,1})|^2 d\mathbf{r}}\right)\right)$$

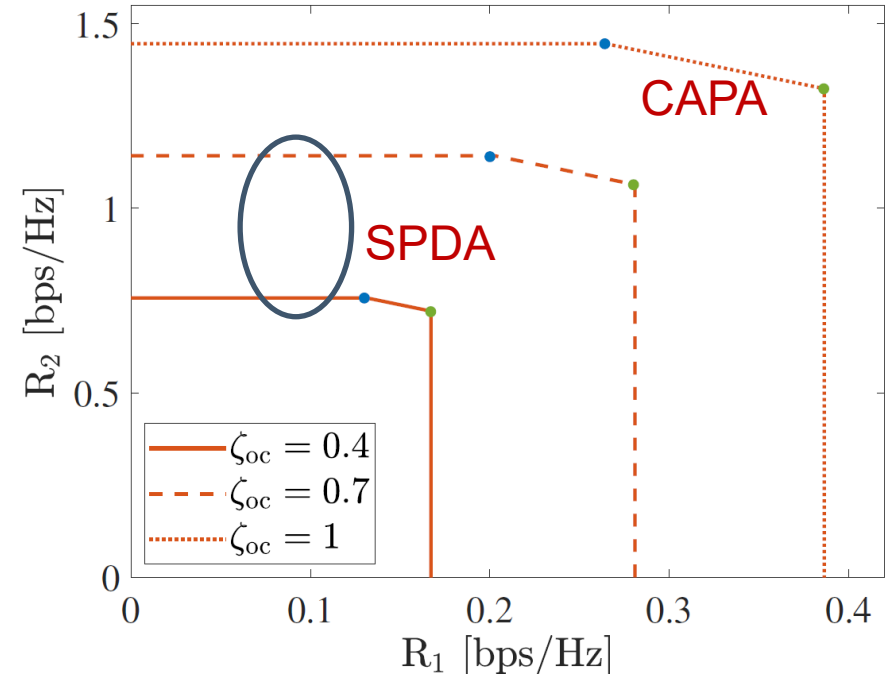
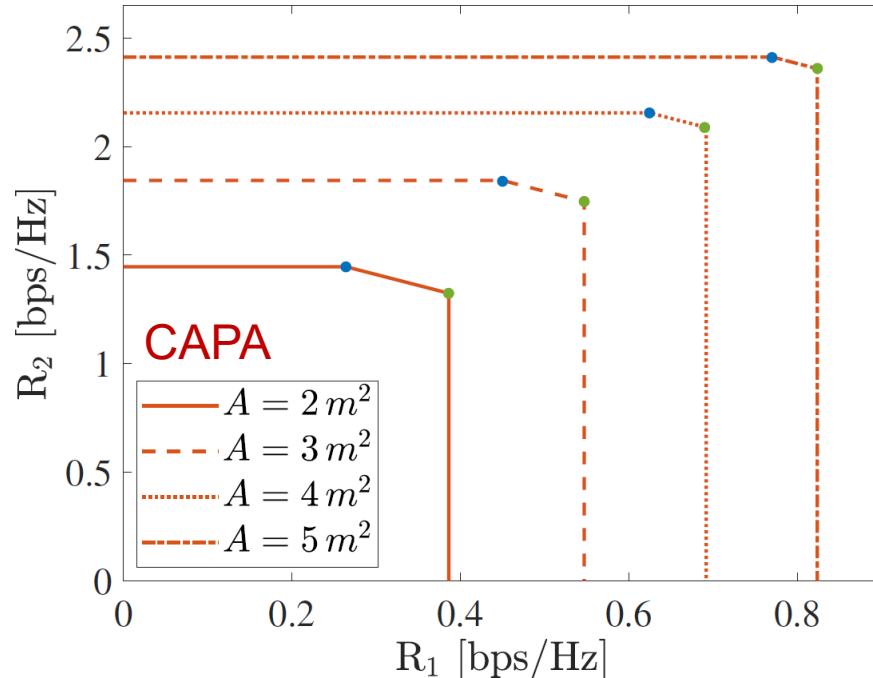
The most challenging task: we do not have matrix inversion, and we have to explicitly calculate the inverse of an operator

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Information-Theoretic Limits for CAPA: Uplink

□ Numerical Results



- Channel capacity region: SIC decoding + time-sharing
- CAPA yields a higher per-user rate and sum-rate than the SPD array
- CAPA yields a larger capacity region than SPD array

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Information-Theoretic Limits for CAPA: Downlink

Multiuser Case with Point-Like Users

$$\underline{Y_{dl,k}(\mathbf{s})} = \underline{s_{dl,k}} \int_{\mathcal{A}} \underline{H(\mathbf{s}_{u,k}, \mathbf{r})} \underline{J_{dl,k}(\mathbf{r})} d\mathbf{r} + \underline{N_{dl,k}(\mathbf{s})} + \sum_{k' \neq k} \underline{s_{dl,k'}} \int_{\mathcal{A}} \underline{H(\mathbf{s}_{u,k'}, \mathbf{r})} \underline{J_{dl,k'}(\mathbf{r})} d\mathbf{r}$$

Received signal Coded data symbol CAPA's aperture User k 's spatial response User k 's source current Noise random field Inter-user interference

Decoding: the SINR of ML decoding at each user

$$\gamma_k = \frac{A_{u,k} \left| \int_{\mathcal{A}} H(\mathbf{s}_{u,k}, \mathbf{r}) J_{dl,k}(\mathbf{r}) d\mathbf{r} \right|^2}{\sigma_k^2 + A_{u,k} \sum_{k' \neq k} \left| \int_{\mathcal{A}} H(\mathbf{s}_{u,k'}, \mathbf{r}) J_{dl,k'}(\mathbf{r}) d\mathbf{r} \right|^2} \Leftarrow \gamma_k = \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sigma_k^2 + \sum_{k' \neq k} |\mathbf{h}_k^H \mathbf{w}_{k'}|^2}$$

Beamforming Design:

Optimization of the Source Currents

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Information-Theoretic Limits for CAPA: Downlink

❑ Multiuser Case Study: Two-User Channel

- Capacity-Achieving Precoding: **dirty-paper coding (DPC)**
- Capacity Characterization: **uplink-downlink duality**

❑ DPC and UL-DL Duality

- DPC: **pre-SIC**, the encoder treats the interference from some user as non-causally known, while its decoder regards the interference from other user as noise.
- UL-DL duality: **fixing the power allocation policy**, the capacity region of a DL channel equals its dual UL channel (with the same spatial response)

❑ UL-DL Duality for CAPA

- UL-DL Transformation: from power allocation to currents $(P_1, P_2) \rightarrow J_{dl,1}(r), J_{dl,2}(r)$
- DL-UL Transformation: from currents to power allocation $J_{dl,1}(r), J_{dl,2}(r) \rightarrow (P_1, P_2)$

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Information-Theoretic Limits for CAPA: Downlink

□ UL-DL Duality-based Capacity Characterization

- DPC order 1 $\rightarrow$ 2: **user 2 will not suffer the IUI from user 1**
- Fixing the allocated power to each user, the per-user rate and sum-rate is the same as that of its dual UL channel with SIC decoding order 2 $\rightarrow$ 1

$$\log_2(1 + \bar{\gamma}_1 \|\mathbf{h}_1\|^2) \Rightarrow \log_2\left(1 + \bar{\gamma}_1 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,1})|^2 d\mathbf{r}\right)$$
$$\log_2(1 + \bar{\gamma}_2 \mathbf{h}_2^H (\mathbf{I} + \bar{\gamma} \mathbf{h}_1 \mathbf{h}_1^H)^{-1} \mathbf{h}_2) \Rightarrow \log_2\left(1 + \bar{\gamma}_2 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,2})|^2 d\mathbf{r} \left(1 - \frac{\bar{\gamma}_1 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,1})|^2 d\mathbf{r} |\rho|^2}{1 + \bar{\gamma}_1 \int_{\mathcal{A}} |\mathbf{H}(\mathbf{r}, \mathbf{s}_{u,1})|^2 d\mathbf{r}}\right)\right)$$

- DPC order 2 $\rightarrow$ 1: **user 1 will not suffer the IUI from user 2**
- Fixing the allocated power to each user, the per-user rate and sum-rate is the same as that of its dual UL channel with SIC decoding order 1 $\rightarrow$ 2

The most important task: finding the details steps for UL-DL transformations under different DPC orders

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Channel Capacity for CAPA: Downlink

❑ Multiuser Case Study: Two-User Channel

- Based on the UL-DL duality, we obtain the sum-rate achieving power allocation and the associated currents from its *dual UL channel* (water-filling method)
- The DL capacity region is the *convex hull* of all its dual UL channels with different power allocation policies

❑ DPC-based DL CAPA Transmission

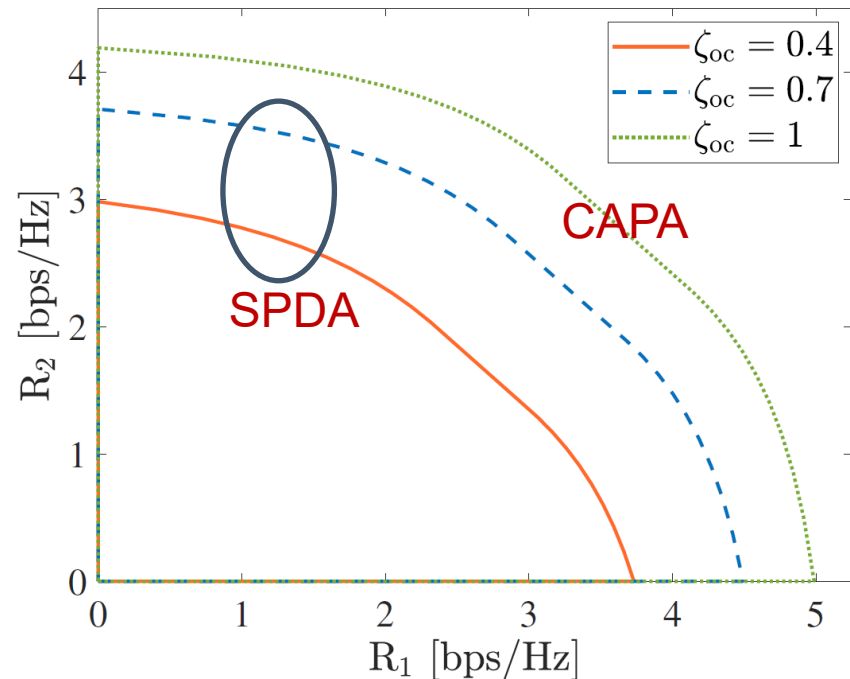
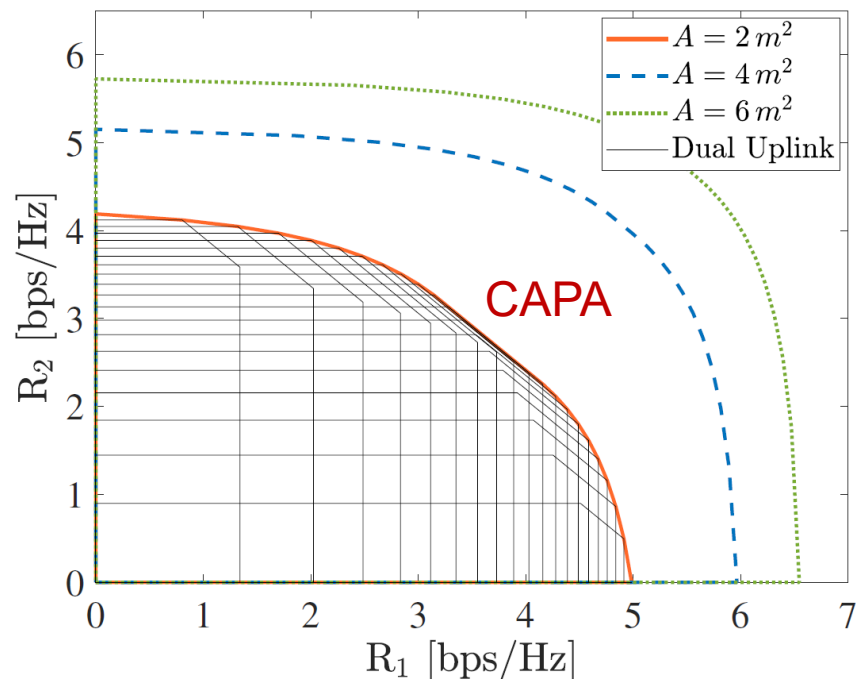
- *Encode the data information into the symbols $s_{dl,1}$, $s_{dl,2}$ using DPC with the DPC order $1 \rightarrow 2$ or the DPC order $2 \rightarrow 1$*
- *Determine the dual power allocation policy P_1 , P_2 according to the specific target quality of service requirements*
- *Generate the source current $J_{dl,1}(r)$, $J_{dl,2}(r)$ based on the UL-DL duality*
- *Formulate the transmit signal $J_{dl,1}(r)s_{dl,1} + J_{dl,2}(r)s_{dl,2}$*

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Information-Theoretic Limits for CAPA: Downlink

□ Numerical Results



- Channel capacity region: **convex hull** of the dual uplink CAPA-based channels
- CAPA yields a higher sum-rate than the SPD array
- CAPA yields a larger capacity region than SPD array

[1] B. Zhao, C. Ouyang, X. Zhang, and Y. Liu, "Continuous aperture array (CAPA)-based wireless communications: Capacity characterization," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2406.15056>



Outline

□ CAPA Basis

- Overview of CAPA (**What**)

- Motivation of Employing CAPA (**Why**)

- Application of CAPA (**How**)

□ Case Study

- Case I: Information-Theoretic Limits for CAPA

- Case II: Performance Analysis for CAPA

- Case III: Beamforming Design for CAPA

- Research Challenges and Opportunities



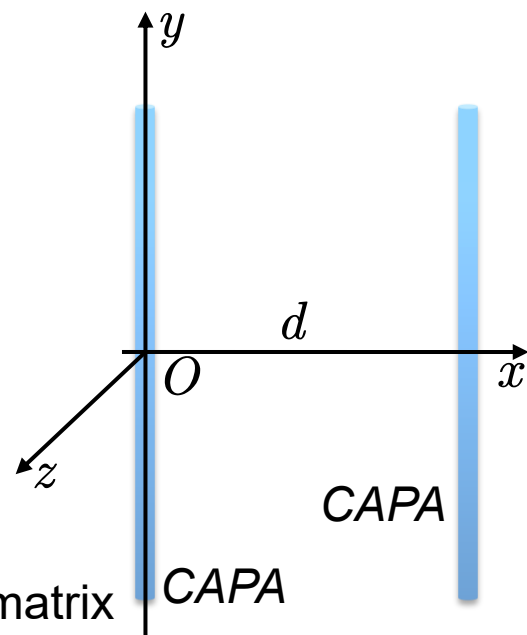
LoS: Degrees-of-Freedom for CAPA

❑ Single-User CAPA-based MIMO

- Tx: a linear CAPA Rx: a linear CAPA
- Channel Model: LoS Model
$$h(\mathbf{r}, \mathbf{s}) \approx \frac{j k_0 \eta e^{-j k_0 \|\mathbf{r} - \mathbf{s}\|}}{4\pi \|\mathbf{r} - \mathbf{s}\|}$$

❑ DoFs of CAPA-based MIMO

- Number of DoFs: number of *parallel sub-channels*
- DoFs of SPD-MIMO: rank of the channel matrix
- Calculating the DoFs of SPD-MIMO: SVD of the channel matrix
- CAPA: “SVD” of the channel response $h(\mathbf{r}, \mathbf{s}) = \sum \sigma_j p_j(\mathbf{r}) q_j(\mathbf{s})$
- CAPA: does a CAPA-MIMO has an infinite number of DoFs?



Calculating the singular value decomposition of a continuous channel response $h(\mathbf{r}, \mathbf{s})$ is a very challenging task.

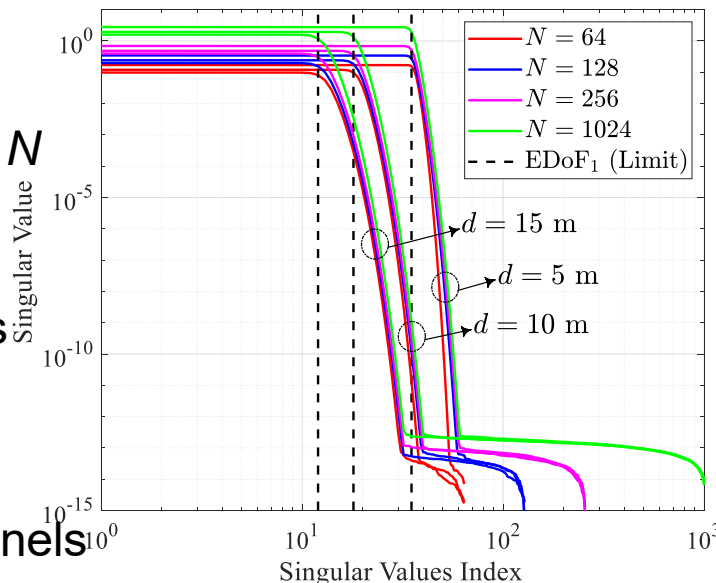
[1] Z. Xie, Y. Liu, J. Xu, X. Wu, and A. Nallanathan “Performance analysis for near-field MIMO: Discrete and continuous aperture antennas,” *IEEE Wireless Commun. Lett.*, vol. 12, no. 12, pp. 2258-2262, Dec. 2023. <https://arxiv.org/abs/2304.06141>



LoS: Degrees-of-Freedom for CAPA

Ordered singular values of $N \times N$ SPD-MIMO

- Tx: a linear SPD array Rx: a linear SPD array
- Fixing the length and increasing antenna number N
- $N \rightarrow \infty$: SPD-MIMO $\Rightarrow$ CAPA-MIMO
- Step-like behavior: *effective DoF/dominate* values



EDoFs of CAPA-MIMO

- Number of EDoFs: number of dominant sub-channels
- Singular values: $\sigma_1 \approx \sigma_2 \approx \dots \approx \sigma_{EDoF} \gg \sigma_{EDoF+1}$
- Approximation of EDoFs: **A Finite Value**

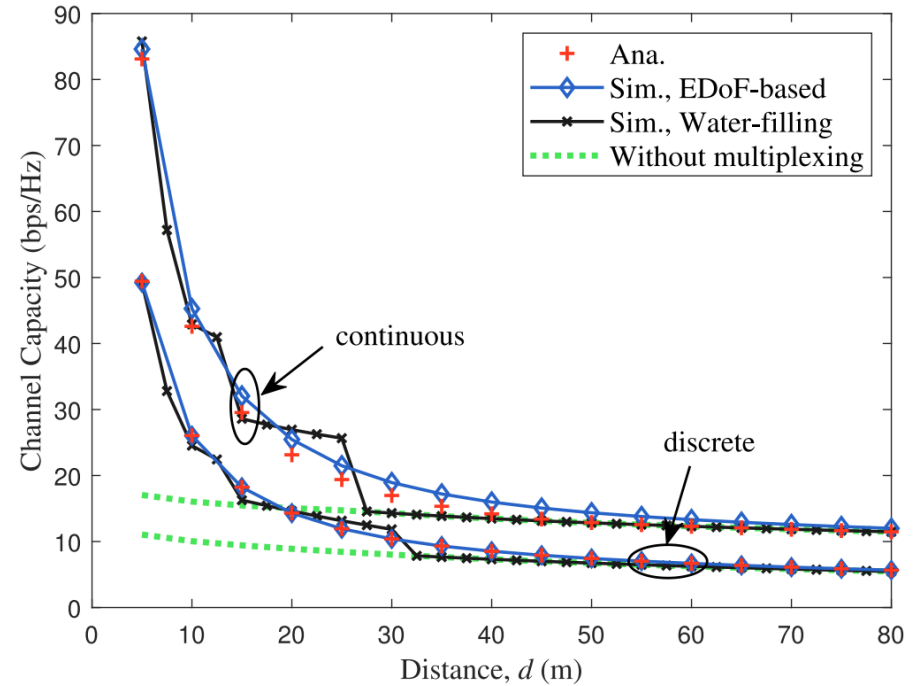
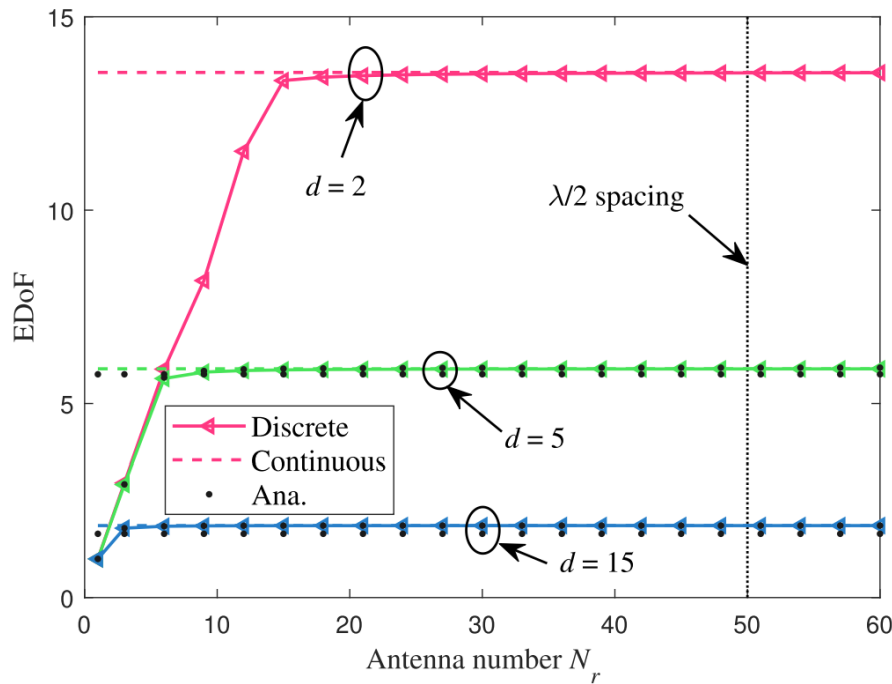
$$EDoF \approx \frac{\left(\int_{\mathcal{A}_t} \int_{\mathcal{A}_r} h^*(\mathbf{r}, \mathbf{s}) h(\mathbf{r}, \mathbf{s}) d\mathbf{r} d\mathbf{s} \right)^2}{\int_{\mathcal{A}_t} \int_{\mathcal{A}_t} K^*(\mathbf{s}, \mathbf{s}') K(\mathbf{s}, \mathbf{s}') d\mathbf{s} d\mathbf{s}'} \quad K(\mathbf{s}, \mathbf{s}') = \int_{\mathcal{A}_r} h^*(\mathbf{r}, \mathbf{s}) h(\mathbf{r}, \mathbf{s}') d\mathbf{r}$$

[1] Z. Xie, Y. Liu, J. Xu, X. Wu, and A. Nallanathan "Performance analysis for near-field MIMO: Discrete and continuous aperture antennas," *IEEE Wireless Commun. Lett.*, vol. 12, no. 12, pp. 2258-2262, Dec. 2023. <https://arxiv.org/abs/2304.06141>



LoS: Degrees-of-Freedom for CAPA

□ Numerical Results



- Fixing the aperture length, SPD-MIMO converges to CAPA-MIMO as N increases
- CAPA yields a larger EDoF and a higher channel capacity than the SPD array
- The channel capacity/EDoF decreases with the propagation distance

[1] Z. Xie, Y. Liu, J. Xu, X. Wu, and A. Nallanathan "Performance analysis for near-field MIMO: Discrete and continuous aperture antennas," *IEEE Wireless Commun. Lett.*, vol. 12, no. 12, pp. 2258-2262, Dec. 2023. <https://arxiv.org/abs/2304.06141>



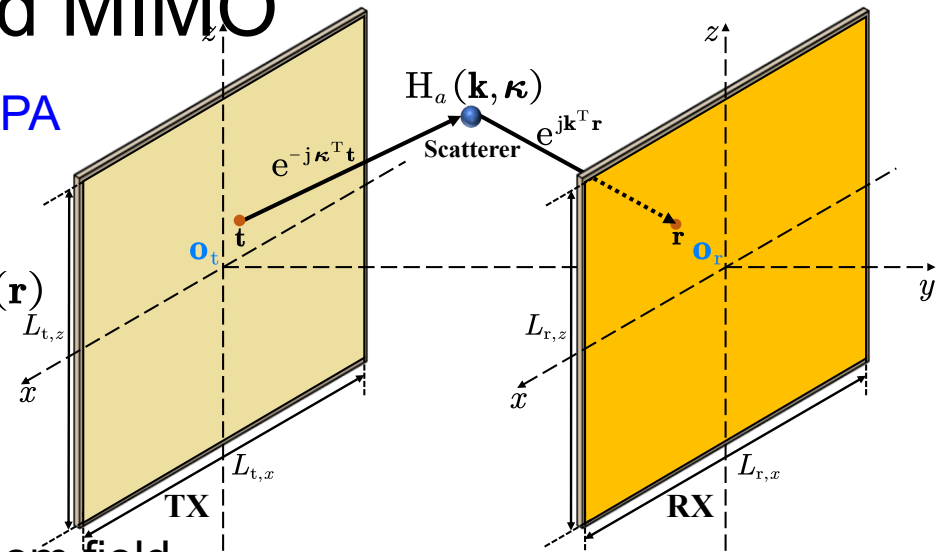
Performance of CAPA with Fading

Single-User CAPA-based MIMO

- Tx: a planar CAPA Rx: a planar CAPA
- Channel model: Rayleigh fading model
- Signal model: $y(\mathbf{r}) = \int_{\mathcal{A}_t} h(\mathbf{r}, \mathbf{t}) s(\mathbf{t}) d\mathbf{t} + n(\mathbf{r})$

Channel Model

- Wavenumber-domain model
- Rayleigh fading: H_a is a Gaussian random field



$$\underbrace{h(\mathbf{r}, \mathbf{t})}_{\text{Spatial-domain channel response}} = \iint_{\mathcal{D}(\boldsymbol{\kappa}) \times \mathcal{D}(\mathbf{k})} \underbrace{e^{j\mathbf{k}^T \mathbf{r}}}_{\text{Receive response}} \underbrace{H_a(\mathbf{k}, \boldsymbol{\kappa})}_{\text{Wavenumber-domain channel response}} \underbrace{e^{-j\boldsymbol{\kappa}^T \mathbf{t}}}_{\text{Transmit response}} d\mathbf{k} d\boldsymbol{\kappa}$$

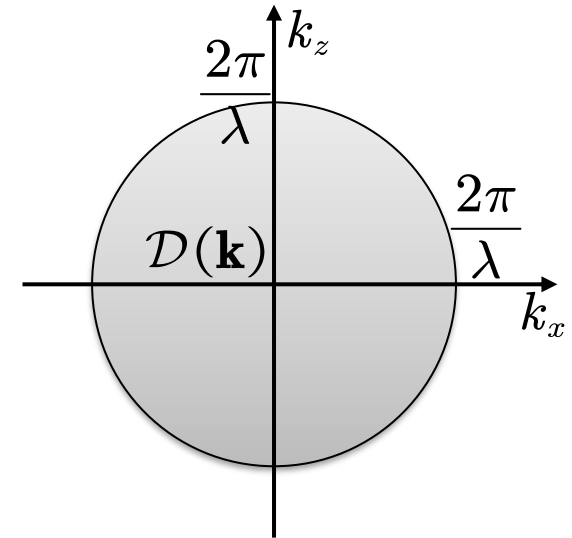
The spatial-domain channel response is the Fourier transformation of the wavenumber-domain channel response

[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Diversity and multiplexing for continuous-aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2408.13948>

Performance of CAPA with Fading

□ Space-Wavenumber Fourier Relationship

- Wavenumber-domain range $D(\mathbf{k}) \times D(\mathbf{k})$
- $D(\mathbf{k}) \times D(\mathbf{k})$ is a two-dimensional finite range
- The spatial-domain response is a *bandlimited* signal
- Nyquist–Shannon theorem: a bandlimited signal can be *information-losslessly* approximated as the linear combination of M orthogonal basis signals
- M : Nyquist sampling rate/DoF
- Nyquist sampling rate for CAPA: $\pi L_x L_z / \lambda^2$ **A Finite Value**
- Approximation of the spatial-domain channel response



$$h(\mathbf{r}, \mathbf{t}) \approx \sum_{n_r=1}^{\pi L_{r,x} L_{r,z} / \lambda^2} \sum_{n_t=1}^{\pi L_{t,x} L_{t,z} / \lambda^2} \underbrace{h_{n_r, n_t}}_{\text{Orthogonal Fourier Bases}} \underbrace{\psi_{n_r}(\mathbf{r}) \phi_{n_t}(\mathbf{t})}_{\text{Gaussian Distributed}}$$

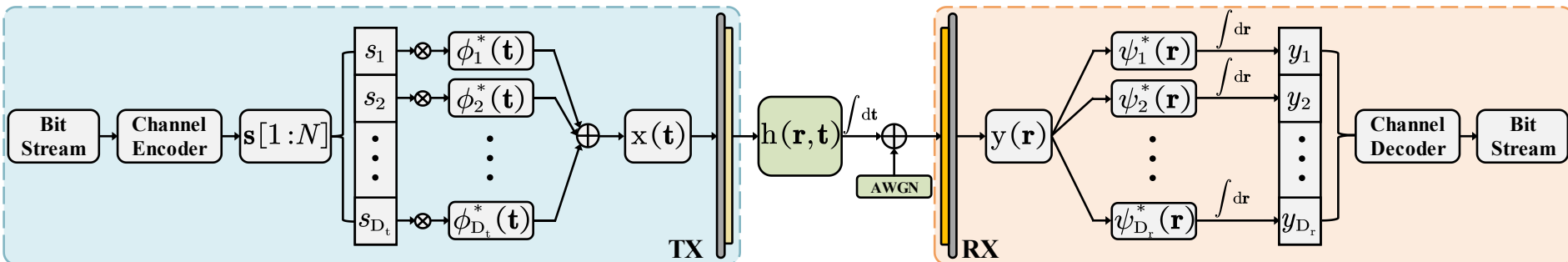
[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Diversity and multiplexing for continuous-aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2408.13948>

Performance of CAPA with Fading

□ Wavenumber-Domain CAPA Transmission

- Using the orthogonality of the Fourier bases
- Transmit side: *mapping information to currents* $s(\mathbf{t}) = \sum n_t s_{n_t} \phi_{n_t}^*(\mathbf{t})$
- Receive side: *using* $\psi_{n_r}^*(\mathbf{r})$ *as equalizers* and then decode the information
- Bit stream $\mapsto$ continuous currents $\mapsto$ received field $\mapsto$ discretized vector $\mapsto$ bit stream
- Continuous $\rightarrow$ Discrete: *the following analyses are same as those for SPD arrays*

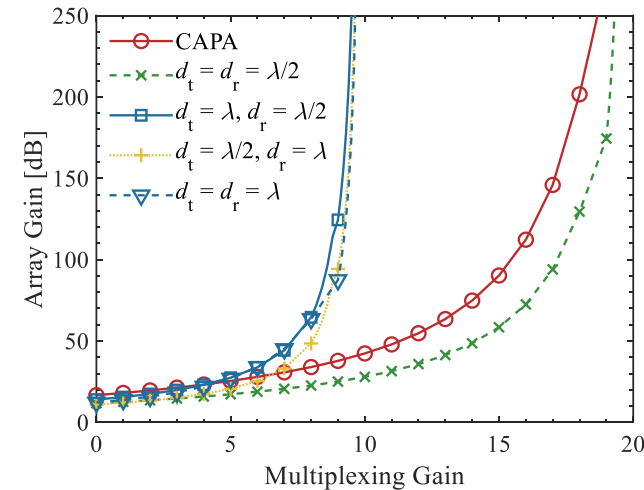
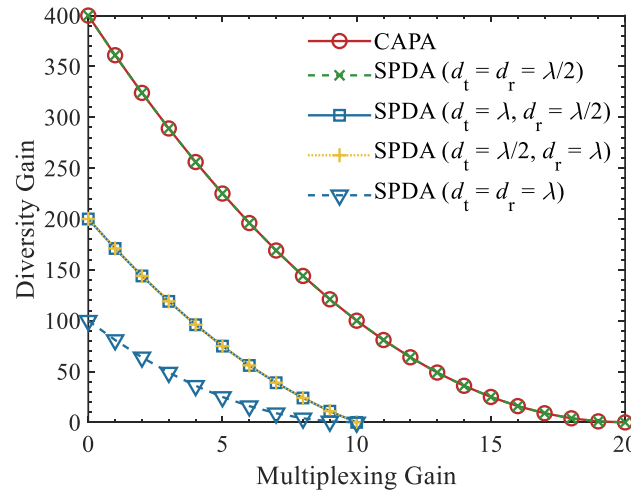
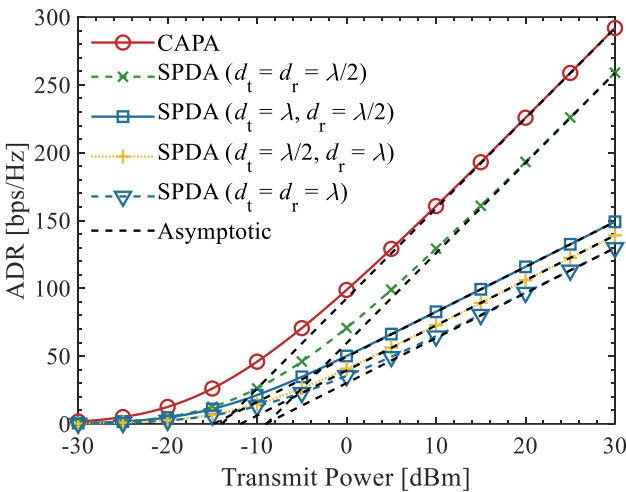
$$y(\mathbf{r}) = \int_{\mathcal{A}_t} h(\mathbf{r}, \mathbf{t}) s(\mathbf{t}) d\mathbf{t} + n(\mathbf{r}) \longrightarrow \mathbf{y} = \mathbf{H}_w \mathbf{s} + \mathbf{n} \implies \text{Channel Capacity: } \log_2 \det(\mathbf{I} + \mathbf{H}_w \mathbf{H}_w^H)$$
- Fading metrics: outage probability, average data rate, diversity-multiplexing tradeoff



[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Diversity and multiplexing for continuous-aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2408.13948>

Performance of CAPA with Fading

□ Numerical Results



- CAPA achieves a higher average data rate compared to SPD arrays
- CAPA attains *the same diversity-multiplexing tradeoff (DMT) as SPD arrays with half-wavelength spacing, while offering a larger array gain*
- CAPA demonstrates a superior DMT compared to SPD arrays with antenna spacings greater than half a wavelength
- In conclusion, CAPA provides a more favorable diversity-multiplexing-array tradeoff than SPD arrays

[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Diversity and multiplexing for continuous-aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2408.13948>



Performance Analysis for CAPA

□ Summary of the Analytical Results

- CAPA outperforms SPD arrays by achieving **a broader capacity region**.
- For LoS channels, CAPA demonstrates **higher channel gain** and supports **a greater number of DoFs** compared to SPD arrays.
- For fading channels, CAPA provides **a superior diversity-multiplexing-array tradeoff** over SPD arrays.

□ Key Takeaways

- Although a CAPA can be roughly treated as an SPD array with an **infinite** number of antennas, its channel gain, EDoFs, diversity gain, multiplexing gain, and overall performance metrics are **finite**.
- The upper limits of these finite values are determined by factors such as the *aperture size*, *operating wavelength*, and the characteristics of the *scattering environment*.

[1] Y. Liu, C. Ouyang, Z. Wang, J. Xu, X. Mu, and Z. Ding, "CAPA: Continuous-aperture arrays for revolutionizing 6G wireless communications," *IEEE Wireless Commun.*, vol. 32, no. 4, pp. 38-45, Aug. 2025. <https://arxiv.org/abs/2412.00894>



Outline

□ CAPA Basis

- Overview of CAPA (**What**)

- Motivation of Employing CAPA (**Why**)

- Application of CAPA (**How**)

□ Case Study

- Case I: Information-Theoretic Limits for CAPA

- Case II: Performance Analysis for CAPA

- Case III: Beamforming Design for CAPA

- Research Challenges and Opportunities



Beamforming Design for CAPA

□ CAPA-based Multi-user System

CAPA: continuous functional problem

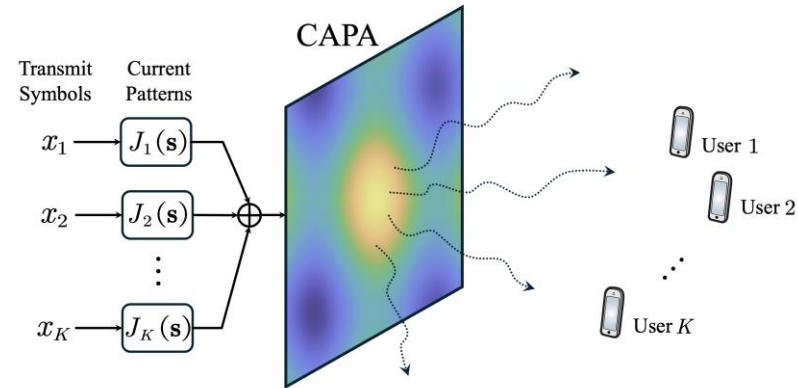
$$\max_{J_k(\mathbf{s})} \sum_{k=1}^K \log_2 \left(1 + \frac{\left| \int_{\mathcal{S}_T} H_k(\mathbf{s}) J_k(\mathbf{s}) d\mathbf{s} \right|^2}{\sum_{i=1, i \neq k}^K \left| \int_{\mathcal{S}_T} H_k(\mathbf{s}) J_i(\mathbf{s}) d\mathbf{s} \right|^2 + \sigma_k^2} \right)$$
$$\text{s.t. } \sum_{k=1}^K \int_{\mathcal{S}_T} |J_k(\mathbf{s})|^2 d\mathbf{s} \leq P_T$$

Continuous Beamformer (Current distribution)

SPD Array: discrete problem

$$\max_{\mathbf{w}_k} \sum_{k=1}^K \log_2 \left(1 + \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{i=1, i \neq k}^K |\mathbf{h}_k^H \mathbf{w}_i|^2 + \sigma_k^2} \right)$$
$$\text{s.t. } \sum_{k=1}^K \|\mathbf{w}_k\|^2 \leq P_T$$

Discrete Beamformer



- CAPA beamforming is hard to be optimized with traditional methods (e.g., CVX)
- Potential methods
 - Discretization method (i.e., sampling and Fourier series)
 - **Calculus of variations method**
 - **Signal subspace method**

Beamforming Design for CAPA

❑ Calculus of variations method

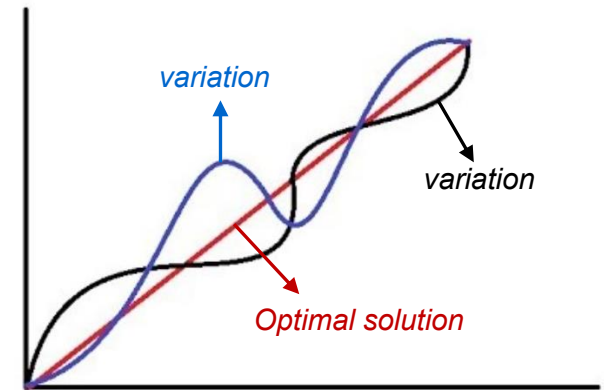
Why

- **Optimality:** discretization method cannot guarantee the optimality since approximation is used
- **Complexity:** discretization method approximate the continuous functions as large-dimensional vectors

Fourier-based discretization: $J_k(\mathbf{s}) = \sum_{n=-\infty}^{\infty} v_{k,n} \Phi_n(\mathbf{s}) \approx \sum_{n=-N}^N v_{k,n} \Phi_n(\mathbf{s})$

How - Calculus of variations

- Calculus of variations is a method from functional analysis that can optimization continuous function without any approximations.



[1] Z. Wang, C. Ouyang, and Y. Liu, "Beamforming optimization for continuous aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun.*, vol. 24, no. 6, pp. 5099-5113, Jun. 2025. <https://arxiv.org/abs/2410.13677>



Introduction to Calculus of Variations

□ What is a variation?

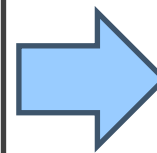
- Relationship between *Variation* and *Derivative*

	Objective function	Optimization variable	Optimality condition
Discrete	A function $f(x)$	Scalar x	Derivative $\frac{df(x)}{dx} = 0$
Continuous	A functional $F(g(x))$	Function $g(x)$	Variation $\delta F(g(x)) = 0$

Variation can be regarded as the derivative of a functional

- Mathematical expression of *Variation*

$$\delta F(g(x)) = \left. \frac{dF(g(x) + \epsilon \eta(x))}{d\epsilon} \right|_{\epsilon=0} = 0$$



Solved via the ***fundamental lemma of calculus of variations***

[1] Z. Wang, C. Ouyang, and Y. Liu, "Beamforming optimization for continuous aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun.*, vol. 24, no. 6, pp. 5099-5113, Jun. 2025. <https://arxiv.org/abs/2410.13677>



Beamforming Design for CAPA

❑ Calculus of variations method

Proposition (Optimal beamforming structure)

Based on calculus of variations, the optimal CAPA beamforming that maximizes the sum-rate has the following structure of Fredholm integral equation of the second kind:

$$J_k(\mathbf{s}) = A_k H_k^*(\mathbf{s}) - \sum_{i=1}^K B_i H_i^*(\mathbf{s}) \int_{S_T} H_i(\mathbf{z}) J_k(\mathbf{z}) d\mathbf{z}$$

- Based on the optimal structure, an *iterative algorithm* is designed for solving the CAPA beamforming problem

- ✓ No approximations – *high quality solution*
- ✓ No need of calculating integrals and large-dimensional vectors in each iteration – *low complexity*

Algorithm 1 BCD-CoV Algorithm for WSR Maximization Problem (11) in CAPA-based Communications

- 1: calculate the channel correlation matrix $\mathbf{H}$
- 2: initialize feasible $J_k^0(\mathbf{s})$, μ_k^0 , and λ_k^0 , and set $n = 0$
- 3: **repeat**
- 4: update $\mathbf{A}^n$ and $\mathbf{B}^n$ by (30) μ_k^n and λ_k^n
- 5: update $\mathbf{W}^{n+1}$ by (32) with $\mathbf{A}^n$ and $\mathbf{B}^n$
- 6: update μ_k^{n+1} and λ_k^{n+1} by (34) and (35) with $\mathbf{A}^n$, $\mathbf{B}^n$, and $\mathbf{W}^{n+1}$
- 7: update $n \leftarrow n + 1$
- 8: **until** the fractional increase of the objective value of problem (15) falls below a threshold
- 9: calculate $J_k(\mathbf{s})$ by (24) with the converged $\mathbf{W}$
- 10: scale $J_k(\mathbf{s})$ according to (14)

[1] Z. Wang, C. Ouyang, and Y. Liu, "Beamforming optimization for continuous aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun.*, vol. 24, no. 6, pp. 5099-5113, Jun. 2025. <https://arxiv.org/abs/2410.13677>



Beamforming Design for CAPA

❑ Correlation-based zero-forcing method

$$\max_{J_k(\mathbf{s})} \sum_{k=1}^K \log_2 \left(1 + \frac{\left| \int_{S_T} H_k(\mathbf{s}) J_k(\mathbf{s}) d\mathbf{s} \right|^2}{\sum_{i=1, i \neq k}^K \left| \int_{S_T} H_k(\mathbf{s}) J_i(\mathbf{s}) d\mathbf{s} \right|^2 + \sigma_k^2} \right)$$

$$\int_{S_T} H_i(\mathbf{s}) J_k(\mathbf{s}) d\mathbf{s} = 0, \forall i \neq k$$

Zero-forcing constraint

Proposition (ZF beamforming)

The CAPA beamformer that achieves the ZF constraint can be expressed as

$$J_k(\mathbf{s}) = \sqrt{\rho_k} J_k^{\text{ZF}}(\mathbf{s}),$$

$$J_k^{\text{ZF}}(\mathbf{s}) = \sum_{j=1}^K u_{k,j} H_j^*(\mathbf{s})$$

- $u_{k,j}$ is the element of matrix $\mathbf{H}^{-1}$
- $\mathbf{H}$ is the channel correlation matrix

$$\mathbf{H}(k, j) = \int_{S_T} H_k(\mathbf{s}) H_j^*(\mathbf{s}) d\mathbf{s}$$

Simple power allocation problem

$$\begin{aligned} & \max_{\{P_k\}_{k=1}^K} \sum_{k=1}^K \alpha_k \log \left(1 + \frac{P_k}{u_{k,k} \sigma_k^2} \right) \\ & \text{s.t. } \sum_{k=1}^K P_k \leq P_T. \end{aligned}$$

Water-filling solution

$$P_k = (\nu \alpha_k - u_{k,k} \sigma_k^2)^+$$

[1] Z. Wang, C. Ouyang, and Y. Liu, "Beamforming optimization for continuous aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun.*, vol. 24, no. 6, pp. 5099-5113, Jun. 2025. <https://arxiv.org/abs/2410.13677>



Beamforming Design for CAPA

□ Performance Comparison

TABLE I: Comparison of Proposed Approaches and Fourier-based Approaches.

Approaches	Approximations Applied?	Initialization Complexity	Optimization Complexity	Beamforming Structure
Proposed CoV-based approach	No	$O(M^2 K^2)$	$O(I_o K^3)$	Optimal
Proposed Corr-ZF approach	No	$O(M^2 K^2)$	$O(K^3) + O(K)$	Asymptotically Optimal
Fourier-based approach	Yes	$O(M^2 K N_F)$	$O(I_o N_F^3)$	Non-Optimal
Fourier-based ZF approach	Yes	$O(M^2 K N_F)$	$O(K^2 N_F) + O(K)$	Non-Optimal

Some typical values: $M \propto 10^1$, $K \propto 10^1$, and $N_F \propto 10^2 \sim 10^4$.

Number of users

Increase exponentially with frequencies and array size

TABLE II: Comparison of Average CPU Time for Different Approaches.

Frequency	$A_T = 0.2 \text{ m}^2$		$A_T = 0.3 \text{ m}^2$		$A_T = 0.4 \text{ m}^2$		$A_T = 0.5 \text{ m}^2$	
	CoV	Fourier	CoV	Fourier	CoV	Fourier	CoV	Fourier
2.4 GHz	0.147 s	0.267 s	0.149 s	0.323 s	0.148 s	0.543 s	0.147 s	0.545 s
7.8 GHz	0.148 s	1.971 s	0.148 s	3.074 s	0.148 s	3.985 s	0.148 s	5.028 s
15 GHz	0.149 s	7.526 s	0.149 s	11.944 s	0.148 s	16.808 s	0.147 s	23.415 s

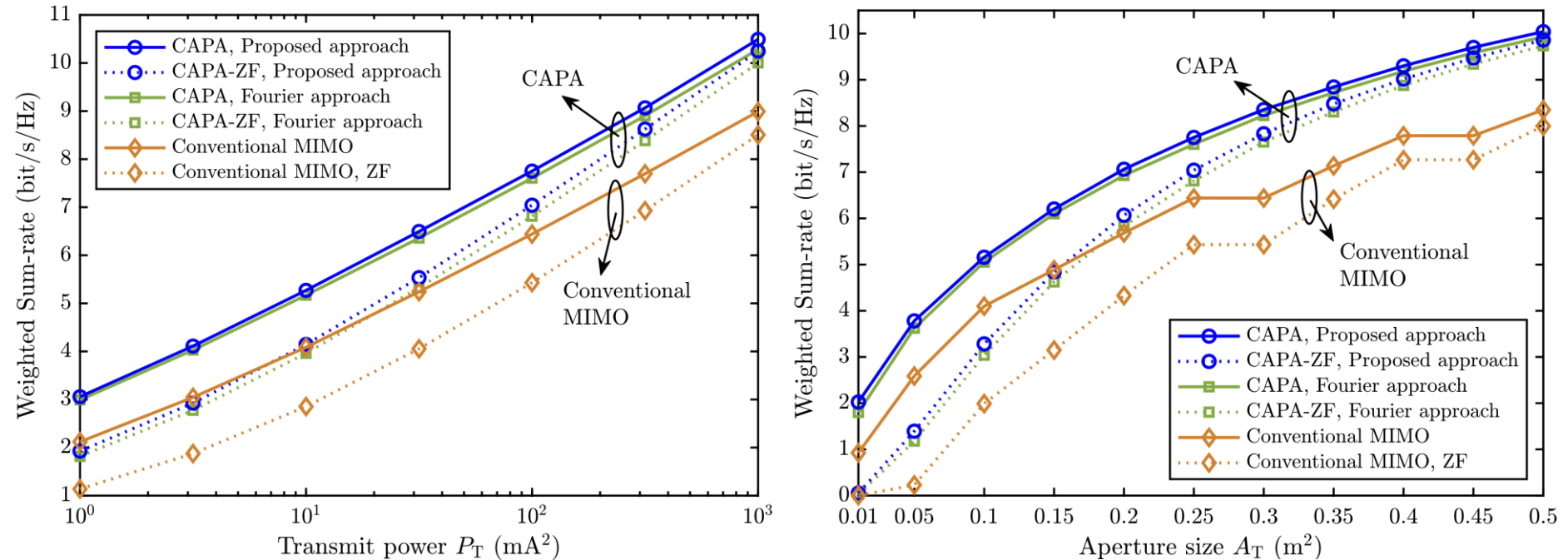
Speed up 160x

[1] Z. Wang, C. Ouyang, and Y. Liu, "Beamforming optimization for continuous aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun.*, vol. 24, no. 6, pp. 5099-5113, Jun. 2025. <https://arxiv.org/abs/2410.13677>



Beamforming Design for CAPA

Performance Comparison



- Proposed methods have *very low complexity* but achieves *better performance* than the Fourier-based discretization method.
- Zero-forcing method is *asymptotically optimal*.

[1] Z. Wang, C. Ouyang, and Y. Liu, "Beamforming optimization for continuous aperture array (CAPA)-based communications," *IEEE Trans. Wireless Commun.*, vol. 24, no. 6, pp. 5099-5113, Jun. 2025. <https://arxiv.org/abs/2410.13677>



Beamforming Design for CAPA

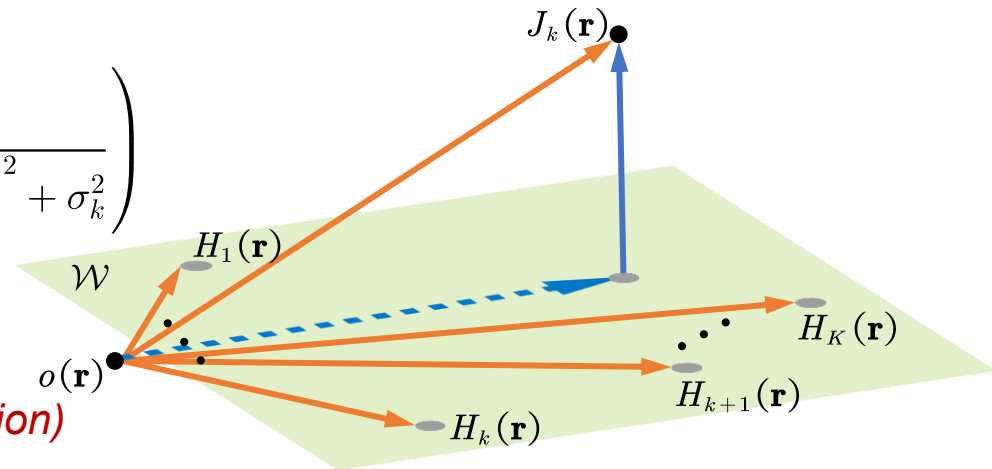
□ Signal subspace method

CAPA: continuous functional problem

$$\max_{J_k(\mathbf{s})} \sum_{k=1}^K \log_2 \left(1 + \frac{\left| \int_{\mathcal{S}_T} H_k(\mathbf{s}) J_k(\mathbf{s}) d\mathbf{s} \right|^2}{\sum_{i=1, i \neq k}^K \left| \int_{\mathcal{S}_T} H_k(\mathbf{s}) J_i(\mathbf{s}) d\mathbf{s} \right|^2 + \sigma_k^2} \right)$$

$$\text{s.t. } \sum_{k=1}^K \int_{\mathcal{S}_T} |J_k(\mathbf{s})|^2 d\mathbf{s} \leq P_T$$

Continuous Beamformer (Current distribution)



Proposition (Signal subspace-based beamforming)

The optimal beamformer shall lie in the signal subspace spanned by each user's spatial response. This means that the optimal current is the weighted sum of these channels.

$$J_k(\mathbf{s}) = \sum_{i=1}^K w_{k,i} H_i(\mathbf{s})$$

[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Linear receive beamforming for CAPA systems," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2411.06620>



香港大學

THE UNIVERSITY OF HONG KONG



Beamforming Design for CAPA

□ Signal subspace method

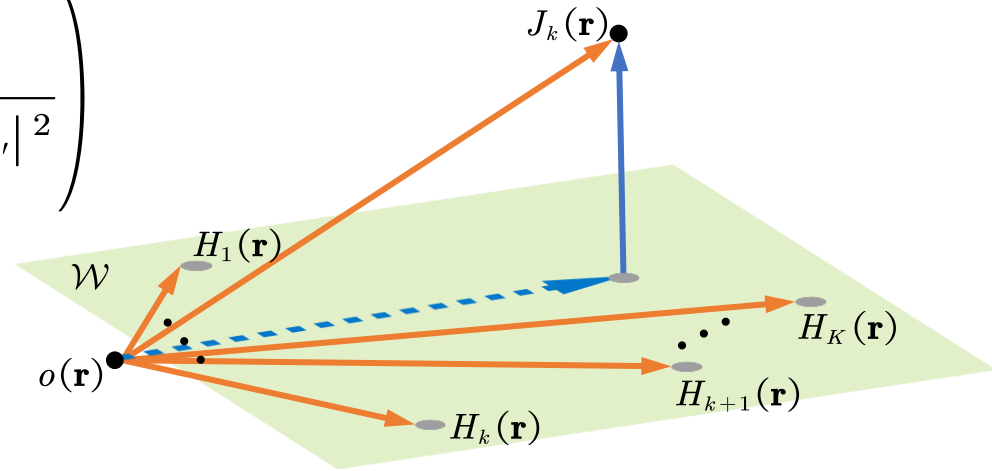
$$\max_{\mathbf{w}_k(\mathbf{s})} \sum_{k=1}^K \alpha_k \log_2 \left(1 + \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sigma_k^2 + \sum_{k' \neq k} |\mathbf{h}_k^H \mathbf{w}_{k'}|^2} \right)$$

$$\text{s.t.} \quad \sum_{k=1}^K \mathbf{w}_k^H \mathbf{H} \mathbf{w}_k \leq P,$$

$$\mathbf{w}_k = [w_{k,1}, \dots, w_{k,K}]^T \quad \mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K]$$

$$[\mathbf{H}]_{i,j} = \int_A h_i^H(\mathbf{s}) h_j(\mathbf{s}) d\mathbf{s}$$

Gram–Schmidt process → Orthonormal Basis of the Subspace



$$\max_{\mathbf{w}_k(\mathbf{s})} \sum_{k=1}^K \alpha_k \log_2 \left(1 + \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sigma_k^2 + \sum_{k' \neq k} |\mathbf{h}_k^H \mathbf{w}_{k'}|^2} \right)$$

$$\text{s.t.} \quad \sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k \leq P,$$

[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Linear receive beamforming for CAPA systems," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2411.06620>

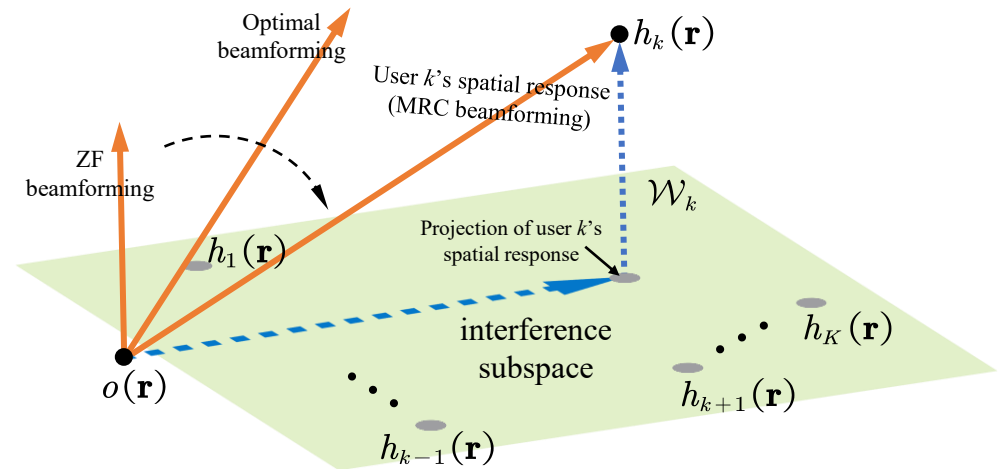
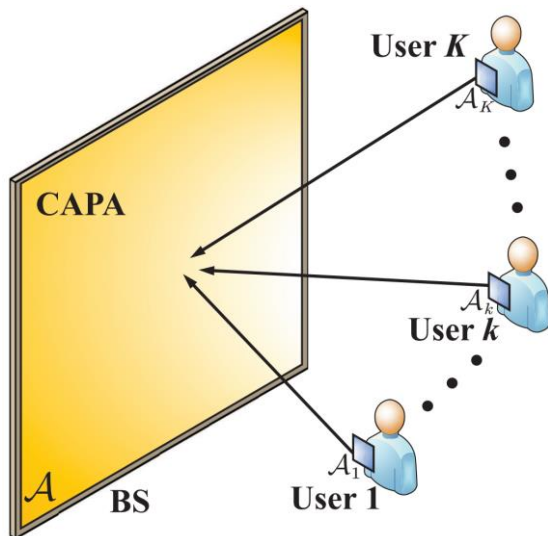


Beamforming Design for CAPA

❑ Signal subspace method: Uplink beamforming design

Type	CAPA		SPDA	
	Beamformer	Complexity	Beamformer	Complexity
MRC	$\mathbf{w}_{\text{MRC}}(\mathbf{r}) = \mathbf{h}(\mathbf{r})\mathbf{I}_K$	$\mathcal{O}(1)$	$\mathbf{W}_{\text{MRC}} = \hat{\mathbf{H}}$	$\mathcal{O}(1)$
ZF	$\mathbf{w}_{\text{ZF}}(\mathbf{r}) = \mathbf{h}(\mathbf{r})\mathbf{R}^{-1}$	$\mathcal{O}(K^3)$	$\mathbf{W}_{\text{ZF}} = \hat{\mathbf{H}}\hat{\mathbf{R}}^{-1}$	$\mathcal{O}(K^3)$
MMSE	$\mathbf{w}_{\text{MMSE}}(\mathbf{r}) = \mathbf{h}(\mathbf{r})(\mathbf{I}_K + \mathbf{P}\mathbf{R})^{-1}$	$\mathcal{O}(K^3)$	$\mathbf{W}_{\text{MMSE}} = \hat{\mathbf{H}}(\mathbf{I}_K + \hat{\mathbf{P}}\hat{\mathbf{R}})^{-1}$	$\mathcal{O}(K^3)$

$$\mathbf{h}(\mathbf{r}) = [h_1(\mathbf{r}), \dots, h_K(\mathbf{r})] \quad \mathbf{w}(\mathbf{r}) = [w_1(\mathbf{r}), \dots, w_K(\mathbf{r})] \quad \hat{\mathbf{H}} = [\mathbf{h}_1, \dots, \mathbf{h}_K]$$

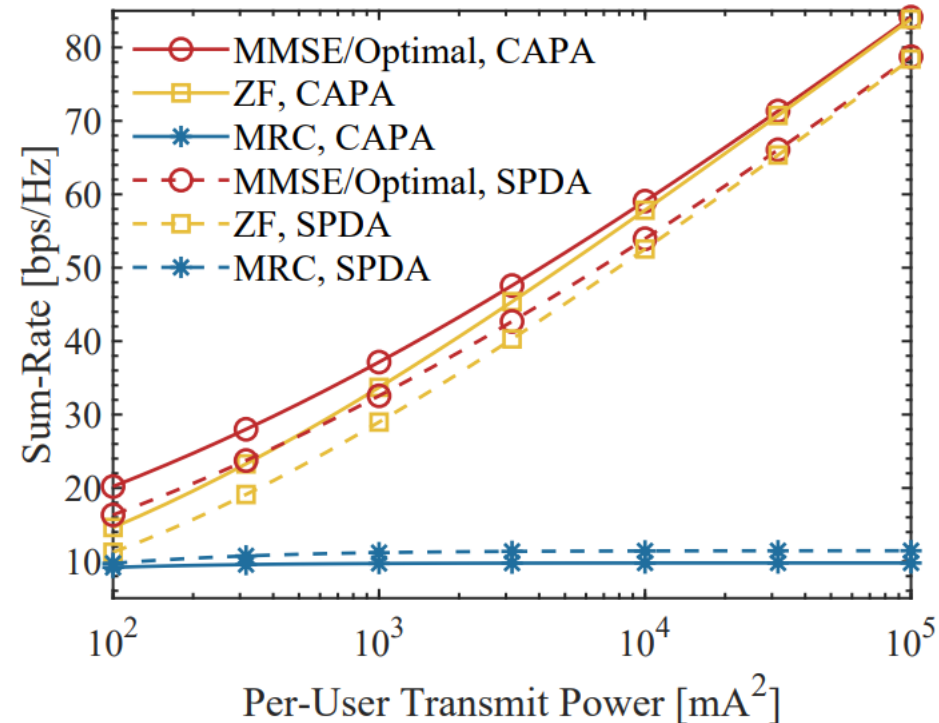
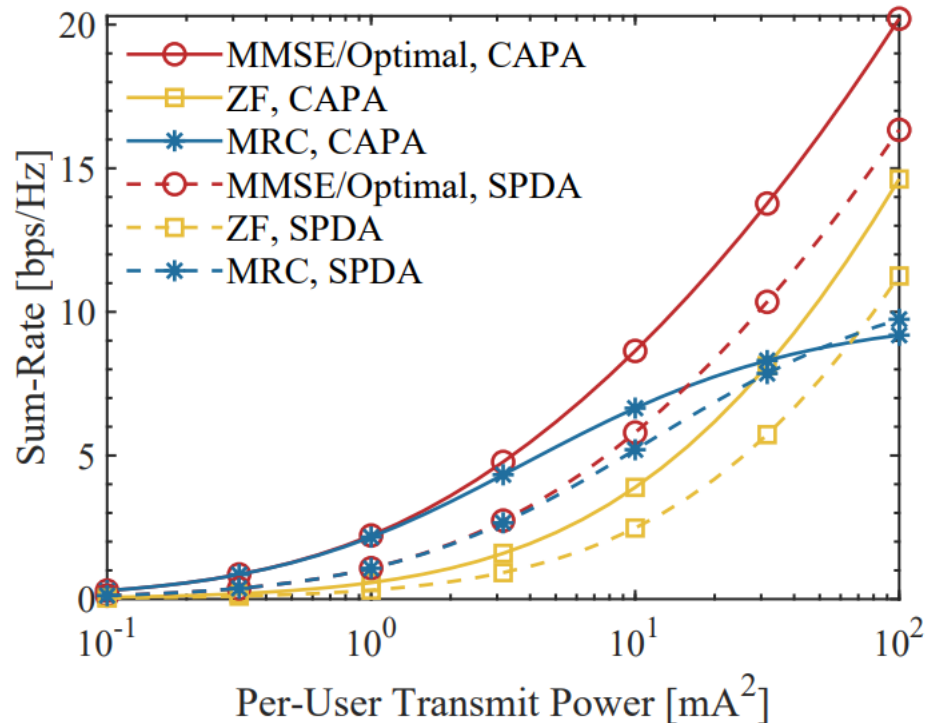


MMSE beamforming is optimal for uplink CAPA communications

[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Linear receive beamforming for CAPA systems," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2411.06620>

Beamforming Design for CAPA

Performance Comparison



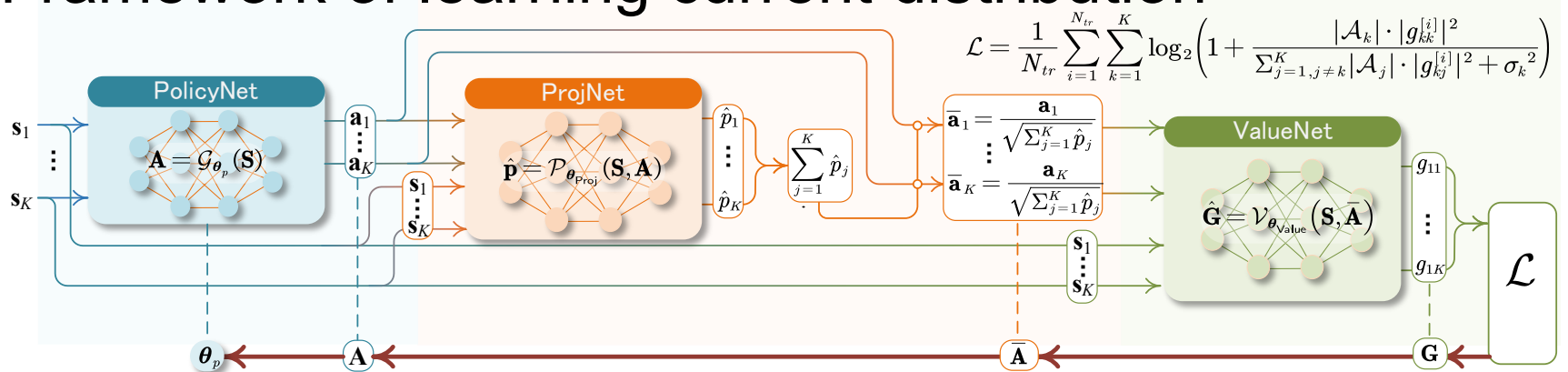
- In the low-SNR regime, MMSE converges to MRC
- In the high-SNR regime, MMSE converges to ZF
- Under ZF and MMSE, CAPAs outperforms SPDAs in terms of the sum-rate
- Under MRC, SPDAs outperform CAPAs in the high-SNR scenarios

[1] C. Ouyang, Z. Wang, X. Zhang, and Y. Liu, "Linear receive beamforming for CAPA systems," *IEEE Trans. Wireless Commun*, Early Access, 2025. <https://arxiv.org/abs/2411.06620>



Learning Current Distribution with DNN

□ Framework of learning current distribution



- Two issues of learning current distribution with traditional DNNs:
- 1. Input $H(\cdot)$ and output $V(\cdot)$ are *functions*, not compatible with DNNs

→ Solution: Find *low-dimensional representations* of functions

□ $H(\cdot)$ only depends on users' positions $\mathbf{s}_k \rightarrow \mathbf{s}_k$ represents $H_k(\cdot)$

□ $V(\cdot)$ spanned by $H(\cdot)$: $V_k(\mathbf{r}) = \sum_{j=1}^K a_{jk} H_j(\mathbf{r}) \rightarrow \mathbf{a}_k = [a_{1k}, \dots, a_{Nk}]^T$ represents $V_k(\cdot)$

Signal subspace method

- 2. Power normalization and loss function with *integrals* → hinder back-propagation

→ Solution: *Learn integrals with DNNs*

[1] J. Guo, Y. Liu, and A. Nallanathan, "Multi-user continuous-aperture array communications: How to learn current distribution?," *IEEE Globecom Workshop*, 2024. <https://arxiv.org/abs/2408.11230>

Learning Current Distribution with DNN

Overall procedure of learning

1. Train ProjNet and ValueNet with supervised learning:

(1) Labels generated by approximating integrals with summations:

$$p_k^{[i]} = \int_{\mathcal{A}} |\mathbf{V}_k^{[i]}(\mathbf{r})| d\mathbf{r} \approx \sum_{m=1}^M |\mathbf{V}_k^{[i]}(\mathbf{r}_m)|^2 \Delta$$

(2) Loss functions:

$$\mathcal{L}(\boldsymbol{\theta}_{\text{Proj}}) = \frac{1}{N_{\text{tr}}} \sum_{i \in S_{\text{Proj}}} \sum_{k=1}^K (\hat{p}_k^{[i]} - p_k^{[i]})^2$$

$$\mathcal{L}(\boldsymbol{\theta}_{\text{Proj}}) = \frac{1}{N_{\text{tr}}} \sum_{i \in S_{\text{Value}}} \sum_{k=1}^K \sum_{j=1}^K (\hat{g}_{kj}^{[i]} - g_{kj}^{[i]})^2$$

Learn by ValueNet

$$\max_{\mathbf{V}_k(\mathbf{r}), k=1, \dots, K} \log_2 \left(1 + \frac{|\mathcal{A}_k| \cdot \left| \int_{\mathcal{A}} \mathbf{H}_k(\mathbf{r}) \mathbf{V}_k(\mathbf{r}) d\mathbf{r} \right|^2}{\sum_{j=1, j \neq k}^K |\mathcal{A}_j| \cdot \left| \int_{\mathcal{A}} \mathbf{H}_k(\mathbf{r}) \mathbf{V}_j(\mathbf{r}) d\mathbf{r} \right|^2} \right)$$

$$\text{s.t. } \sum_{k=1}^K \int_{\mathcal{A}} |\mathbf{V}_k(\mathbf{r})|^2 d\mathbf{r} = P_{\text{max}}$$

Learn by ProjNet

2. Train PolicyNet with unsupervised learning:

(1) Forward propagation:

Power normalization: $\bar{\mathbf{V}}_k(\mathbf{r}) = \mathbf{V}_k(\mathbf{r}) \sqrt{P_{\text{max}}} / \sqrt{\sum_{j=1}^K \int_{\mathcal{A}} |\mathbf{V}_j(\mathbf{r})|^2 d\mathbf{r}}$

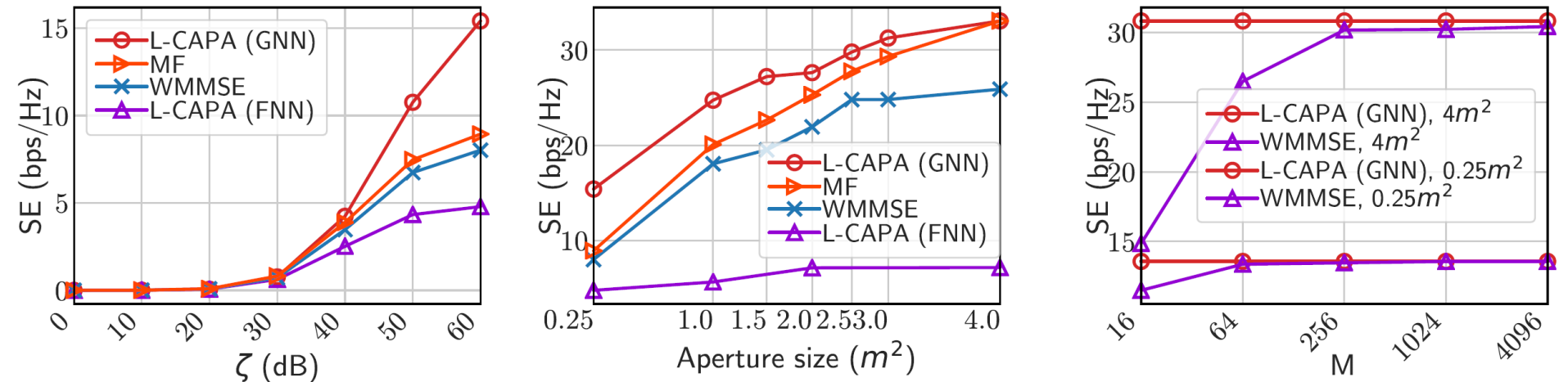
Loss function: $\mathcal{L} = \frac{1}{N_{\text{tr}}} \sum_{i=1}^{N_{\text{tr}}} \sum_{k=1}^K \log_2 \left(1 + \frac{|\mathcal{A}_k| \cdot \left| \int_{\mathcal{A}} \mathbf{H}_k^{[i]}(\mathbf{r}) \bar{\mathbf{V}}_k^{[i]}(\mathbf{r}) d\mathbf{r} \right|^2}{\sum_{j=1, j \neq k}^K |\mathcal{A}_j| \cdot \left| \int_{\mathcal{A}} \mathbf{H}_k^{[i]}(\mathbf{r}) \bar{\mathbf{V}}_j^{[i]}(\mathbf{r}) d\mathbf{r} \right|^2} \right)$

Computed by ValueNet

(2) Back propagation: $\boldsymbol{\theta}_p \leftarrow \boldsymbol{\theta}_p - \phi \frac{\partial \mathcal{L}(\boldsymbol{\theta}_p)}{\partial \boldsymbol{\theta}_p}$

Learning Current Distribution with DNN

Simulation Results



- Left: SE under different SNR (ζ) $\rightarrow$ Higher gain in interference-limited system
- Middle: SE under different aperture size $\rightarrow$ Higher gain with lower aperture
- Right: learned current distribution can achieve upper bound of numerical algorithm under discrete antenna system when the number of antennas $\rightarrow \infty$

[1] J. Guo, Y. Liu, and A. Nallanathan, "Multi-user continuous-aperture array communications: How to learn current distribution?," *IEEE Globecom Workshop*, 2024. <https://arxiv.org/abs/2408.11230>



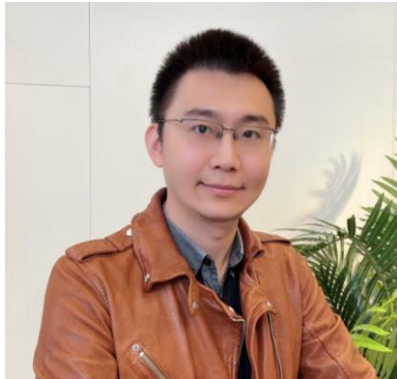
Thanks for your attention Q & A



香 港 大 學

THE UNIVERSITY OF HONG KONG

Tech-driven academic-industry collaboration



Prof. Yuanwei Liu
Co-Director



Prof. Kaibin Huang
Co-Director

Joint research projects

Focused on themes such as edge AI inference, 6G architecture, distributed training, and LLM deployment on low-power edge devices.

Custom graduate programs

Industry-driven MSc/PhD tracks with joint supervision, real-world project topics, and embedded internship opportunities.

Executive training & certification

Short courses and workshops taught by HKU faculty tailored to enterprise upskilling needs.

Visiting scholars & expert lectures

Faculty and researchers will engage with partners on-site for short-term consulting and technical support.

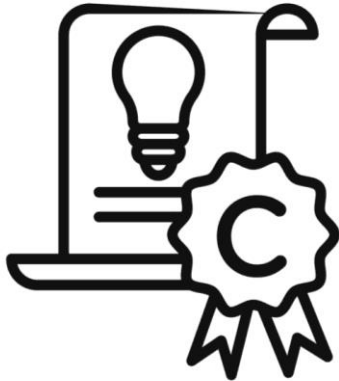


IP Policy & Membership Models

Leveraged Research



Companies can gain non-exclusive, royalty-free, non-transferable access to shareable IPs and outputs from HKU's government-funded projects, subject to internal approval, without incurring additional cost.



Sponsored Research



- **Gift Projects:** Typically do not generate or transfer IP. Suitable for open-ended exploration and student sponsorship.
- **Contract Projects:** IP is typically co-owned between the company and HKU, with commercial licensing terms negotiated through the Technology Transfer Office (TTO).
- **Named Laboratories:** Long-term collaborations with joint visibility and customized IP arrangements, generally following contract project policies.



How to join?

For more information about 6G@HKU:

<https://www.eee.hku.hk/~yuanwei/6G-HKU.html>



香港大學

THE UNIVERSITY OF HONG KONG

