Comments on "Class of Cyclic-Based Estimators for Frequency-Offset Estimation of OFDM Systems"

Yik-Chung Wu and Erchin Serpedin

***Abstract*—This comment corrects several errors found in the paper, "Class of Cyclic-Based Estimators for Frequency-Offset Estimation of OFDM Systems." In addition, we show that the minimum variance unbiased estimator for frequency offset derived in the above paper is the maximum-likelihood estimator when the timing delay is perfectly known.**

I. System Model

First, the probability density function (pdf) of the observation vector in [1, eq. (3)] is for a real-valued observation vector and a negative sign is missing inside the exponential function. Since the observation vector is complex-valued, the correct pdf should be [2, pp. 507]

$$p(\mathbf{x}, \vartheta, \varepsilon) = \frac{1}{\pi^{2N+L}\det(\mathbf{R})}\exp[-\mathbf{x}^{*}\mathbf{R}^{-1}\mathbf{x}]. \qquad (1)$$

Second, in [1, eqs. (10) and (12)], the factor 1/2 inside the exponential function should not be present. This can be easily proved by careful derivation of [1, eqs. (10) and (12)]. Although this constant factor does not affect the maximum-likelihood (ML) and minimum variance unbiased (MVU) estimators, this factor does affect the expression of the Cramer–Rao lower bound (CRLB). This point would be further detailed in Section IV.

II. ML Estimator

The ML estimator for the timing delay ϑ should be given by

$$\vartheta_{ML} = \arg\max_{\vartheta}\{2\left|T_1(\mathbf{x}, \vartheta)\right| - |a|T_2(\mathbf{x}, \vartheta)\} \qquad (2)$$

rather than [1, eq. (15)]. This can be proved by careful mathematical derivations. Furthermore, the authors in [1] claim that their ML estimator is an improved version of the results in [3] by including the extra case $N+1 \leq \vartheta \leq N+L$. By noting [3, eq. (12)], one can see that the ML estimator has to be in the form of (2) in this comment; otherwise, the ML estimator in the above paper does not reduce to the results presented in [3] if the extra case $N+1 \leq \vartheta \leq N+L$ is ignored.

Paper approved by S. K. Wilson, the Editor for Coding Theory and Applications of the IEEE Communications Society. Manuscript received April 12, 2004; revised September 21, 2004 and November 12, 2004.

The authors are with the Department of Electrical Engineering, Texas A&M University, College Station, TX 77843-3128 USA (e-mail: ycwu@ee.tamu.edu; serpedin@ee.tamu.edu).

Digital Object Identifier 10.1109/TCOMM.2005.843444

III. MVU Estimator

In equation (19) of [1], an unbiased estimator for ε is presented [based on the second-moment expression (18)]. The correct expression for [1, eq. (19)] should be

$$\breve{\varepsilon} = -\frac{1}{2\pi}\Im\left\{\ln\left[\frac{1}{L\sigma_s^2}T_1(\mathbf{x}, \vartheta)\right]\right\} \qquad (3)$$

where, in addition to the replacement of the summation with $T_1(\mathbf{x}, \vartheta)$ to include $1 \leq \vartheta \leq N$ and $N+1 \leq \vartheta \leq N+L$ cases, a negative sign is added, since it should appear, in view of [1, eq. (18)].

Furthermore, even with the above modifications, $\breve{\varepsilon}$ is only an asymptotically unbiased estimator for ε. This is because, in general, the expectation operation cannot commute with the nonlinear logarithm, that is

$$\mathbb{E}(\breve{\varepsilon}) \neq -\frac{1}{2\pi}\Im\left\{\ln\left[\frac{1}{L\sigma_s^2}\mathbb{E}\left(T_1(\mathbf{x}, \vartheta)\right)\right]\right\}. \qquad (4)$$

However, in asymptotic cases (when the signal-to-noise ratio (SNR) is high and/or L is large), the variation of $T_1(\mathbf{x}, \vartheta)$ is very small, so that a linear approximation of the logarithm can be applied, and the locations of the expectation operation and logarithm can be interchanged. Therefore, all the derivations for the MVU estimator afterwards are valid only asymptotically.

One additional comment on the MVU estimator for ε is as follows. Noting that

$$\begin{aligned}\breve{\varepsilon} &= -\frac{1}{2\pi}\Im\left\{\ln\left[\frac{1}{L\sigma_s^2}T_1(\mathbf{x}, \vartheta)\right]\right\} && (5)\\ &= -\frac{1}{2\pi}\Im\left\{\ln\left[T_1(\mathbf{x}, \vartheta)\right]\right\} && (6)\\ &= -\frac{1}{2\pi}\angle T_1(\mathbf{x}, \vartheta) && (7)\end{aligned}$$

which is equivalent to the ML estimator for ε if the timing delay ϑ is perfectly known. However, it is well-known that the ML estimator is asymptotically efficient [2, p. 167] (i.e., the ML estimator is asymptotically unbiased and with the smallest possible variance). Therefore, no new result has been reported in the above paper in reference to the MVU estimator.

IV. CRLB

As discussed in Section I of this comment, an extra factor of 1/2 is included in the exponential functions in the pdf expressions of [1, eqs. (10) and (12)]. Removing this factor and recalculating the CRLB leads to

$$\mathrm{CRLB} = \frac{\left(1+\frac{1}{\mathrm{SNR}}\right)^2 - 1}{2(2\pi)^2 L}. \qquad (8)$$

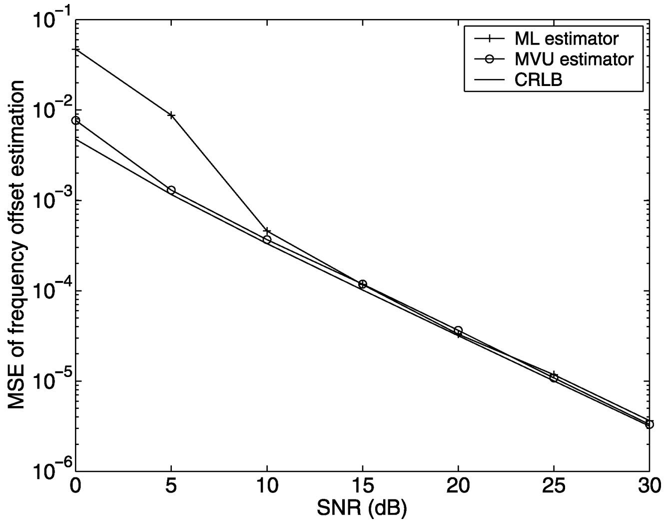

Fig. 1. Performance comparison between the ML estimator, the MVU estimator, and the CRLB.

In Fig. 1, we provide a simple simulation example to illustrate the performance differences between the ML estimator (which jointly estimate the unknown timing delay ϑ and frequency offset ε), the MVU estimator (which assumes ϑ perfectly known and only ε is estimated), and the CRLB. All the simulation conditions are the same as those in [1], except that the estimators are based on the correct equations presented in this comment. It can be seen from Fig. 1 that the ML estimator approaches the CRLB at high SNRs. Actually, the SNR threshold for the ML estimator is just about 10 dB, which shows that even without the knowledge of timing offset ϑ, the ML estimator can still perform pretty well for SNR $\geq$ 10 dB.

V. Moment-Based Estimator

Equation (38) in [1] is actually the first-order Taylor expansion of $T_3(\mathbf{x})$, and not that of $\hat{\varepsilon}_{mom}$. Furthermore, [1, eqs. (39) and (40)] are the expressions for the variance of $T_3(\mathbf{x})$, and not that of $\hat{\varepsilon}_{mom}$.

References

[1] N. Lashkarian and S. Kiaei, "Class of cyclic-based estimators for frequency-offset estimation of OFDM systems," *IEEE Trans. Commun.*, vol. 48, pp. 2139–2149, Dec. 2000.

[2] S. M. Kay, *Fundamentals of Statistical Signal Processing—Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[3] J.-J. van de Beek, M. Sandell, and P. O. Borjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Trans. Signal Process.*, vol. 45, pp. 1800–1805, Jul. 1997.