Signal Processing ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at SciVerse ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Cooperative beamforming for dual-hop amplify-and-forward multi-antenna relaying cellular networks

Chengwen Xing [a,*], Shaodan Ma [b], Minghua Xia [c], Yik-Chung Wu [d]

[a] School of Information and Electronics, Beijing Institute of Technology, Beijing, China
[b] Department of Electrical and Computer Engineering, University of Macau, Macao
[c] King Abdullah University of Science and Technology (KAUST), Saudi Arabia
[d] Department of Electrical and Electronic Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong

ARTICLE INFO

Article history:
Received 18 July 2011
Received in revised form
19 January 2012
Accepted 26 April 2012

Keywords:
Amplify-and-forward (AF)
Cellular network
Multiple-input multiple-output (MIMO)
Minimum mean-square-error
Relay station

ABSTRACT

In this paper, linear beamforming design for amplify-and-forward relaying cellular networks is considered, in which base station, relay station and mobile terminals are all equipped with multiple antennas. The design is based on minimum mean-square-error criterion, and both uplink and downlink scenarios are considered. It is found that the downlink and uplink beamforming design problems are in the same form, and iterative algorithms with the same structure can be used to solve the design problems. For the specific cases of fully loaded or overloaded uplink systems, a novel algorithm is derived and its relationships with several existing beamforming design algorithms for conventional MIMO or multiuser systems are revealed. Simulation results are presented to demonstrate the performance advantage of the proposed design algorithms.

1. Introduction

Cooperative communication is a promising technology to improve quality and reliability of wireless links [1–20]. One of the most important application scenarios of cooperative communications is cellular network. Due to shadowing or deep fading of wireless channels, base station may not be able to sufficiently cover all mobile terminals in a cell, especially those on the edge. Deployment of relay stations is an effective and economic way to improve the communication quality in cellular networks, as shown in Fig. 1.

In cooperative cellular networks, there are two major strategies in relaying. Relay station can either decode the received signal before retransmission [21] or simply amplify-and-forward (AF) the received signal to the corresponding destination without decoding [22]. AF strategy has low complexity and minimal processing delay, and is more secure. These reasons make AF preferable for practical applications. In fact, deployment of AF relay station with multiple antennas to enlarge coverage of base station has been adopted as a standard transmission technique in a lot of communication systems, e.g., LTE, IMT-Advanced and Winner Project [23,24].

Beamforming design in cooperative networks, which refers to the design of beamforming matrices at the sources, relays and destinations, has attracted considerable research interest recently, due to its potential of significant performance improvement. Comparing with traditional beamforming designs for point-to-point MIMO systems without relay stations [30–37] the beamforming design for AF relaying systems is significantly different and more challenging due to the cumulative noise effect from multiple hops and the fact that the noise at the relay will also be amplified and forwarded to the destination

* Corresponding author. Tel.: +86 15801543420.
E-mail addresses: xingchengwen@gmail.com (C. Xing), shaodanma@umac.mo (S. Ma), minghua.xia@kaust.edu.sa (M. Xia), ycwu@eee.hku.hk (Y.-C. Wu).

0165-1684/$ - see front matter Crown Copyright © 2012 Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sigpro.2012.04.017

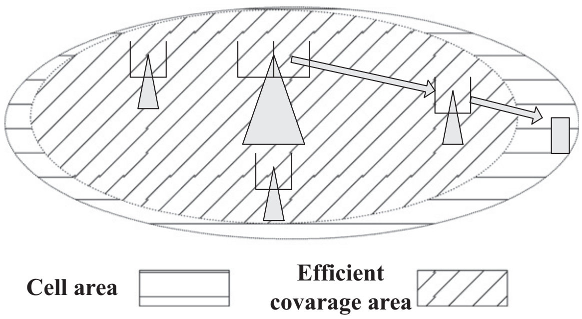

Fig. 1. Amplify-and-forward MIMO relaying cellular network.

node [17,18]. It is thus nontrivial to carefully investigate the beamforming designs for AF relaying systems.

There is a rich literature about beamforming design for cooperative networks. Generally speaking, the beamforming design heavily depends on network topologies, design objectives and constraints. It would significantly change from system to system. From the objective point of view, the beamforming design can be basically classified into two categories. One is to maximize capacity [25], while the other is to minimize the mean-square error (MSE) of the recovered data [26]. The MSE criterion has been widely chosen since it aims at recovering the data as accurately as possible. It has been extensively adopted in traditional point-to-point [27–29] or multiuser MIMO systems [30–36,46,47]. The MSE minimization is also related to capacity maximization [29,34] if a suitable weighting is applied to different data streams.

According to the network considered, the beamforming design problem can be further classified into the following categories. The first one considers a simple and fundamental network with one source, one destination and one relay. Various beamforming design algorithms have been proposed to achieve different objectives under different constraints [7,5,40]. For example, in our previous work [40], a joint design of the relay beamforming matrix and the destination equalizer is proposed to achieve minimum MSE of the received signals without considering the design of source beamformer. The second category focuses on a network with one source, multiple distributed single antenna relays and one destination [8–10,17,18,28]. Usually this kind of design is not preferable for cellular applications [23,24] due to high overhead requirement for the purpose of synchronization among multiple relays. The third category investigates a recently proposed two-way relaying network in which two sources exchange information with the help of one relay using physical layer network coding [13–16]. Perfect synchronization between two sources is required in order to achieve the throughput improvement promised by the two-way relaying strategy, which thus limits its application in cellular networks. The fourth category considers a cellular network with multiple single antenna mobile terminals, one multiple antenna relay and one multiple antenna base station. This relay cellular network has been investigated from various points of view. For example, beamforming design for capacity maximization has been considered for the downlink in [21], and quality-of-service based power control has been investigated for the downlink in [22]. However, in the next generation multi-media wireless communications, it is likely that the size of a mobile terminal allows multiple antennas to be deployed. Unfortunately, extension from the previous works on single antenna mobile terminals to multi-antenna terminals is by no mean straightforward.

In this paper, we take a step forward to investigate a cellular AF cooperative network with multiple mobile terminals, one relay and one base station, each node equipped with multiple antennas. In particular, we consider the joint design of the precoders, forwarding matrix, and equalizers for both uplink and downlink under individual power constraints. The design problems are formulated as optimization problems with the objective of minimizing the sum MSE of multiple detected data streams. Since extension of the presented algorithm to weighted MSE criterion is straightforward, we focus on sum MSE criterion for notational clarity. The contributions of the paper are listed as follows. Firstly, the downlink is considered and an iterative algorithm is proposed to jointly design the precoder at base station, forwarding matrix at relay station and equalizers at mobile terminals. Secondly, for the uplink case, we demonstrate that the formulation of the beamforming design problem has the same form as that in the downlink, and the same iterative algorithm can be applied. Thirdly, since the proposed general iterative solution provides little insight and has high complexity, we derive another low complexity algorithm for the uplink under a special case when the number of independent data streams from different mobile terminals is greater than or equal to their number of antennas. It is found that the resultant solution includes several existing algorithms for multiuser MIMO or AF relay network with single antenna as special cases.

For clarity, we would like to further point out the differences between this work and our previous work in [40].

(a) Without channel estimation errors, the proposed algorithm in [40] is exactly a special case of the uplink transceiver design in this work. Notice that in [40], there is only one source node and the source precoder design is not considered. In this work, there are multiple source nodes and the source precoder design is taken into account. For AF MIMO relaying systems, source precoder design is of great importance and offers additional design freedom for performance improvement. This fact has been revealed in [5].

(b) In the uplink case, multiple sources impose individual transmit power constraints on the beamforming design, which is different from that with one total power constraint in [40] and makes the design challenging.

(c) Beamforming design for the downlink is also discussed here. Since the signals are distributedly estimated at multiple destination nodes, the beamforming design is more challenging than its counterpart in uplink.

The paper is organized as follows. In Section 2, beamforming design problem for the downlink is investigated, and an iterative algorithm is presented. In Section 3, the analogy of the uplink and downlink beamforming design problems is demonstrated. Furthermore, another low complexity beamforming design algorithm is derived for the special case of fully loaded or overloaded system, and the relationships of

Please cite this article as: C. Xing, et al., Cooperative beamforming for dual-hop amplify-and-forward multi-antenna relaying cellular networks, Signal Processing (2012), http://dx.doi.org/10.1016/j.sigpro.2012.04.017

this algorithm with other existing algorithms are discussed. Simulation results are given in Section 4 to demonstrate the effectiveness of the proposed algorithms. Finally, conclusions are drawn in Section 5.

The following notations are used throughout this paper. Boldface lowercase letters denote vectors, while boldface uppercase letters denote matrices. The notation $\mathbf{Z}^{\mathrm{H}}$ denotes the Hermitian of the matrix $\mathbf{Z}$, and $\mathrm{Tr}(\mathbf{Z})$ is the trace of the matrix $\mathbf{Z}$. The symbol $\mathbf{I}_M$ denotes an $M \times M$ identity matrix, while $\mathbf{0}_{M,N}$ denotes an $M \times N$ all zero matrix. The notation $\mathbf{Z}^{1/2}$ is the Hermitian square root of the positive semidefinite matrix $\mathbf{Z}$, such that $\mathbf{Z}^{1/2}\mathbf{Z}^{1/2} = \mathbf{Z}$ and $\mathbf{Z}^{1/2}$ is also a Hermitian matrix. The operation $\mathrm{diag}\{[\mathbf{A}\ \mathbf{B}]\}$ is defined as a block diagonal matrix with $\mathbf{A}$ and $\mathbf{B}$ as block diagonal. The symbol $\mathbb{E}\{\bullet\}$ represents the statistical expectation. The operation $\mathrm{vec}(\mathbf{Z})$ stacks the columns of the matrix $\mathbf{Z}$ into a single vector. The symbol $\otimes$ denotes the Kronecker product. For two Hermitian matrices, $\mathbf{C} \succcurlyeq \mathbf{D}$ means that $\mathbf{C}-\mathbf{D}$ is a positive semidefinite matrix.

2. Downlink beamforming design

2.1. System model and problem formulation

On the boundary of a cell, due to shadowing or deep fading, the direct link between base station (BS) and mobile terminals may not be good enough to maintain normal communication. Then mobile terminals can rely on relay station to communicate with BS. As shown in Fig. 2a, in downlink, signal is first transmitted from the BS to the relay station and then the relay station forwards the received signal to the corresponding mobile terminals. It is assumed that the BS has N_B antennas and the relay station has N_R antennas. For the kth mobile terminal, it has $N_{M,k}$ antennas. The BS needs to simultaneously communicate with K mobile terminals via a single relay station. There are L_k data streams to be transmitted from the BS to the kth mobile terminal, and the signal for the kth mobile terminal is denoted by a $L_k \times 1$ vector $\mathbf{s}_k$. It is assumed that different data streams are independent, i.e., $\mathbb{E}\{\mathbf{s}_k\mathbf{s}_j^{\mathrm{H}}\} = \mathbf{0}_{L_k,L_j}$ when $k \neq j$ and $\mathbb{E}\{\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}\} = \mathbf{I}_{L_k}$. With separate precoder $\mathbf{T}_k$ for different mobile terminals, the received signal at the relay station is

$$\mathbf{r} = \mathbf{H}_{BR}\mathbf{T}\mathbf{s} + \boldsymbol{\eta}, \tag{1}$$

where $\mathbf{H}_{BR}$ denotes the $N_R \times N_B$ channel matrix between the BS and relay station, $\mathbf{T} = [\mathbf{T}_1, \ldots, \mathbf{T}_K]$, $\mathbf{s} = [\mathbf{s}_1^{\mathrm{T}}, \ldots, \mathbf{s}_K^{\mathrm{T}}]^{\mathrm{T}}$ and the vector $\boldsymbol{\eta}$ denotes the additive Gaussian noise with zero mean and covariance matrix $\mathbf{R}_{\boldsymbol{\eta}}$. The power constraint at the BS is given by $\sum_k \mathrm{Tr}(\mathbf{T}_k\mathbf{T}_k^{\mathrm{H}}) \leq P_s$, where P_s is the maximum transmit power.

At the relay station, before retransmission the signal $\mathbf{r}$ is multiplied with a forwarding matrix $\mathbf{W}$ under a power constraint $\mathrm{Tr}(\mathbf{W}\mathbf{R}_{\mathbf{r}}\mathbf{W})^{\mathrm{H}} \leq P_r$, where P_r is the maximum transmit power at the relay station and $\mathbf{R}_{\mathbf{r}}$ is the covariance matrix of the received signal $\mathbf{r}$

$$\mathbf{R}_{\mathbf{r}} = \mathbf{H}_{BR}\mathbf{T}\mathbf{T}^{\mathrm{H}}\mathbf{H}_{BR}^{\mathrm{H}} + \mathbf{R}_{\boldsymbol{\eta}}. \tag{2}$$

Finally, at the kth mobile terminal, the received signal $\mathbf{y}_k$ is

$$\mathbf{y}_k = \mathbf{H}_{RM,k}\mathbf{W}\mathbf{H}_{BR}\mathbf{T}\mathbf{s} + \mathbf{H}_{RM,k}\mathbf{W}\boldsymbol{\eta} + \mathbf{v}_k, \tag{3}$$

where matrix $\mathbf{H}_{RM,k}$ is the $N_{M,k} \times N_R$ channel matrix between the relay station and the kth mobile terminal, and $\mathbf{v}_k$ is the additive Gaussian noise at the kth mobile terminal with zero mean and covariance matrix $\mathbf{R}_{\mathbf{v}_k}$.

At each mobile terminal, an equalizer $\mathbf{G}_k$ is employed to detect the data. The mean-square-error (MSE) of data detection at the kth terminal is

$$\mathrm{MSE}_k(\mathbf{G}_k, \mathbf{W}, \mathbf{T}_k) = \mathbb{E}\{\|\mathbf{G}_k\mathbf{y}_k - \mathbf{s}_k\|^2\}, \tag{4}$$

$$\begin{aligned}\mathrm{MSE}_k(\mathbf{G}_k, \mathbf{W}, \mathbf{T}_k) = {} & \mathrm{Tr}(\mathbf{G}_k(\mathbf{H}_{RM,k}\mathbf{W}\mathbf{R}_{\mathbf{r}}\mathbf{W}^{\mathrm{H}}\mathbf{H}_{RM,k}^{\mathrm{H}} + \mathbf{R}_{\mathbf{v}_k})\mathbf{G}_k^{\mathrm{H}}) \\ & - \mathrm{Tr}(\mathbf{G}_k\mathbf{H}_{RM,k}\mathbf{W}\mathbf{H}_{BR}\mathbf{T}_k) \\ & - \mathrm{Tr}((\mathbf{G}_k\mathbf{H}_{RM,k}\mathbf{W}\mathbf{H}_{BR}\mathbf{T}_k)^{\mathrm{H}}) + \mathrm{Tr}(\mathbf{I}_{L_k}).\end{aligned} \tag{5}$$

Now defining $\mathbf{y} = [\mathbf{y}_1^{\mathrm{T}}, \ldots, \mathbf{y}_K^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{H}_{RM} = [\mathbf{H}_{RM,1}^{\mathrm{T}}, \ldots, \mathbf{H}_{RM,K}^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{v} = [\mathbf{v}_1^{\mathrm{T}}, \ldots, \mathbf{v}_K^{\mathrm{T}}]^{\mathrm{T}}$, and $\mathbf{G} = \mathrm{diag}\{[\mathbf{G}_1, \ldots, \mathbf{G}_K]\}$, the sum MSE can be written as

$$\begin{aligned}\mathrm{MSE}_D(\mathbf{G}, \mathbf{W}, \mathbf{T}) = {} & \sum_{k=1}^{K} \mathrm{MSE}_k(\mathbf{G}_k, \mathbf{W}, \mathbf{T}_k) \\ = {} & \mathrm{Tr}(\mathbf{G}(\mathbf{H}_{RM}\mathbf{W}\mathbf{R}_{\mathbf{r}}\mathbf{W}^{\mathrm{H}}\mathbf{H}_{RM}^{\mathrm{H}} + \mathbf{R}_{\mathbf{v}})\mathbf{G}^{\mathrm{H}}) \\ & - \mathrm{Tr}(\mathbf{G}\mathbf{H}_{RM}\mathbf{W}\mathbf{H}_{BR}\mathbf{T}) \\ & - \mathrm{Tr}((\mathbf{G}\mathbf{H}_{RM}\mathbf{W}\mathbf{H}_{BR}\mathbf{T})^{\mathrm{H}}) + \mathrm{Tr}(\mathbf{I}_L),\end{aligned} \tag{6}$$

where $L = \sum_{k=1}^{K} L_k$ and $\mathbf{R}_{\mathbf{v}} = \mathrm{diag}\{[\mathbf{R}_{\mathbf{v}_1}, \ldots, \mathbf{R}_{\mathbf{v}_K}]\}$.

Therefore, the downlink beamforming optimization problem can be formulated as

$$\begin{aligned}& \min_{\mathbf{G},\mathbf{W},\mathbf{T}} \quad \mathrm{MSE}_D(\mathbf{G}, \mathbf{W}, \mathbf{T}) \\ & \text{s.t.} \quad \mathrm{Tr}(\mathbf{T}\mathbf{T}^{\mathrm{H}}) \leq P_s, \\ & \mathrm{Tr}(\mathbf{W}\mathbf{R}_{\mathbf{r}}\mathbf{W}^{\mathrm{H}}) \leq P_r, \\ & \mathbf{G} = \mathrm{diag}\{[\mathbf{G}_1, \ldots, \mathbf{G}_K]\}.\end{aligned} \tag{7}$$

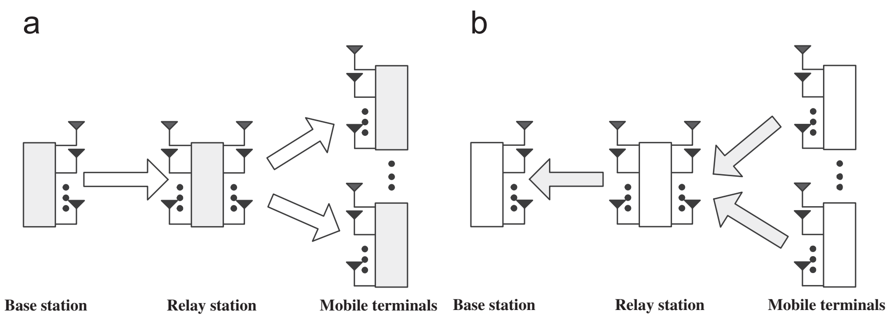

Fig. 2. Amplify-and-forward MIMO relaying downlink and uplink cellular systems. (a) Downlink case and (b) Uplink case.

Please cite this article as: C. Xing, et al., Cooperative beamforming for dual-hop amplify-and-forward multi-antenna relaying cellular networks, Signal Processing (2012), http://dx.doi.org/10.1016/j.sigpro.2012.04.017

Note that the optimization problem (7) is different from that in [40] where no source precoder and no block diagonal constraint on the equalizers $\mathbf{G}$ are included. It thus calls for a different solution. It is clear that the optimization problem (7) is a nonconvex optimization problem for $\mathbf{T}$, $\mathbf{W}$ and $\mathbf{G}$, and there is no closed-form solution. This challenge remains even for the special case of multiuser MIMO systems [30,31,35] where only single hop transmission is involved. However, notice that when two out of the three variables are fixed, the optimization problem (7) for the remaining variable is a convex problem, and thus can be solved. Therefore, an iterative algorithm alternating the design of three variables can be employed.

2.2. Proposed iterative algorithm

(1) *Equalizer design at the destination*: When $\mathbf{T}$ and $\mathbf{W}$ are fixed, the optimization problem (7) is an unconstrained convex quadratic optimization problem for $\mathbf{G}$. Furthermore, since the structure of $\mathbf{G}$ is block diagonal, the design of individual $\mathbf{G}_k$ are decoupled. Therefore, the necessary and sufficient condition for the optimal solution is

$$\frac{\partial \sum_k \mathrm{MSE}_k(\mathbf{G}_k,\mathbf{W},\mathbf{T}_k)}{\partial \mathbf{G}_k^*} = \mathbf{0}_{L_k,N_{M,k}}, \tag{8}$$

and the optimal equalizer for the kth mobile terminal can be easily shown to be

$$\mathbf{G}_k = (\mathbf{H}_{RM,k}\mathbf{W}\mathbf{H}_{BR}\mathbf{T}_k)^{\mathrm{H}}(\mathbf{H}_{RM,k}\mathbf{W}\mathbf{R}_{\mathbf{r}}\mathbf{W}^{\mathrm{H}}\mathbf{H}_{RM,k}^{\mathrm{H}} + \mathbf{R}_{\mathbf{v}_k})^{-1}. \tag{9}$$

(2) *Forwarding matrix design at the relay station*: When $\mathbf{T}$ and $\mathbf{G}$ are fixed, the optimization problem (7) is a constrained convex optimization problem for the variable $\mathbf{W}$, and the Karush–Kuhn–Tucker (KKT) conditions are the necessary and sufficient conditions for the optimal solution [38]. The KKT conditions of the optimization problem (7) with respective to $\mathbf{W}$ are [39]

$$\mathbf{H}_{RM}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{H}_{RM}\mathbf{W}\mathbf{R}_{\mathbf{r}} + \lambda\mathbf{W}\mathbf{R}_{\mathbf{r}} = (\mathbf{H}_{BR}\mathbf{T}\mathbf{G}\mathbf{H}_{RM})^{\mathrm{H}}, \tag{10}$$

$$\lambda(\mathrm{Tr}(\mathbf{W}\mathbf{R}_{\mathbf{r}}\mathbf{W}^{\mathrm{H}}) - P_r) = 0, \quad \lambda \geq 0, \tag{11}$$

$$\mathrm{Tr}(\mathbf{W}\mathbf{R}_{\mathbf{r}}\mathbf{W}^{\mathrm{H}}) \leq P_r, \tag{12}$$

where λ is the Lagrange multiplier.

Based on the first KKT condition (10), the optimal forwarding matrix $\mathbf{W}$ can be written as

$$\mathbf{W} = (\mathbf{H}_{RM}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{H}_{RM} + \lambda\mathbf{I})^{-1}(\mathbf{H}_{BR}\mathbf{T}\mathbf{G}\mathbf{H}_{RM})^{\mathrm{H}}\mathbf{R}_{\mathbf{r}}^{-1}, \tag{13}$$

where the value of λ is computed using (11) and (12). Since λ also appears in $\mathbf{W}$, (11) and (12) depend on λ in a nonlinear way and there is no closed-form solution, but it can be computed using a simple bisection search proposed in [40].

(3) *Precoder design at the BS*: When $\mathbf{W}$ and $\mathbf{G}$ are fixed, the optimization problem (7) can be reformulated as the following convex quadratic optimization problem for the precoder $\mathbf{T}$:

$$\begin{aligned}\min_{\mathbf{T}} \quad & \mathrm{Tr}(\mathbf{N}_0^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{A}_0\mathbf{T}\mathbf{N}_0) + 2\mathcal{R}\{\mathrm{Tr}(\mathbf{B}_0^{\mathrm{H}}\mathbf{T})\} + c_0 \\ \text{s.t.} \quad & \mathrm{Tr}(\mathbf{N}_1^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{A}_1\mathbf{T}\mathbf{N}_1) + 2\mathcal{R}\{\mathrm{Tr}(\mathbf{B}_1^{\mathrm{H}}\mathbf{T})\} + c_1 \leq 0, \\ & \mathrm{Tr}(\mathbf{N}_2^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{A}_2\mathbf{T}\mathbf{N}_2) + 2\mathcal{R}\{\mathrm{Tr}(\mathbf{B}_2^{\mathrm{H}}\mathbf{T})\} + c_2 \leq 0,\end{aligned} \tag{14}$$

where the corresponding parameters are defined as

$$\begin{aligned}&\mathbf{A}_0 = \mathbf{H}_{BR}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{H}_{RM}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{H}_{RM}\mathbf{W}\mathbf{H}_{BR}, \quad \mathbf{A}_1 = \mathbf{I}, \\ &\mathbf{A}_2 = \mathbf{H}_{BR}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{H}_{BR}, \\ &\mathbf{B}_0^{\mathrm{H}} = -\mathbf{G}\mathbf{H}_{RM}\mathbf{W}\mathbf{H}_{BR}, \quad \mathbf{B}_1 = \mathbf{B}_2 = \mathbf{0}, \\ &\mathbf{N}_0 = \mathbf{N}_1 = \mathbf{N}_2 = \mathbf{I}_L, \\ &c_0 = \mathrm{Tr}(\mathbf{R}_{\boldsymbol{\eta}}\mathbf{W}^{\mathrm{H}}\mathbf{H}_{RM}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{H}_{RM}\mathbf{W}) + \mathrm{Tr}(\mathbf{I}_L) + \mathrm{Tr}(\mathbf{G}\mathbf{R}_{\mathbf{v}}\mathbf{G}^{\mathrm{H}})), \\ &c_1 = -P_s, \quad c_2 = \mathrm{Tr}(\mathbf{W}\mathbf{R}_{\boldsymbol{\eta}}\mathbf{W}^{\mathrm{H}}) - P_r.\end{aligned} \tag{15}$$

Notice that the objective function and the constraints are of the same form. Using the property $\mathrm{Tr}(\mathbf{A}\mathbf{B}) = \mathrm{vec}^{\mathrm{H}}(\mathbf{A}^{\mathrm{H}})\mathrm{vec}(\mathbf{B})$ and the property of Kronecker product, we can write ($l = 0,1,2$)

$$\begin{aligned}\mathrm{Tr}(\mathbf{N}_l^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{A}_l\mathbf{T}\mathbf{N}_l) &= \mathrm{Tr}(\mathbf{N}_l^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{A}_l^{\mathrm{H}/2}\mathbf{A}_l^{1/2}\mathbf{T}\mathbf{N}_l) \\ &= \mathrm{vec}^{\mathrm{H}}(\mathbf{A}_l^{1/2}\mathbf{T}\mathbf{N}_l)\mathrm{vec}(\mathbf{A}_l^{1/2}\mathbf{T}\mathbf{N}_l) \\ &= \mathrm{vec}^{\mathrm{H}}(\mathbf{T})(\mathbf{N}_l^* \otimes \mathbf{A}_l^{\mathrm{H}/2})(\mathbf{N}_l^{\mathrm{T}} \otimes \mathbf{A}_l^{1/2})\mathrm{vec}(\mathbf{T}),\end{aligned} \tag{16}$$

where the first equality is based on the fact that $\mathbf{A}_l$'s are positive semidefinite matrices. Furthermore, we can also write $\mathrm{Tr}(\mathbf{B}_l^{\mathrm{H}}\mathbf{T}) = \mathrm{vec}^{\mathrm{H}}(\mathbf{B}_l^{\mathrm{H}})\mathrm{vec}(\mathbf{T})$. Putting these two results into (14) and introducing an auxiliary variable t [41], the problem (14) is equivalent to the following optimization problem:

$$\begin{aligned}\min_{\mathbf{T},t} \quad & t \\ \text{s.t.} \quad & \mathrm{vec}^{\mathrm{H}}(\mathbf{T})(\mathbf{N}_0^* \otimes \mathbf{A}_0^{\mathrm{H}/2})(\mathbf{N}_0^{\mathrm{T}} \otimes \mathbf{A}_0^{1/2})\mathrm{vec}(\mathbf{T}) \\ & \quad \leq t - 2\mathcal{R}\{\mathrm{vec}^{\mathrm{H}}(\mathbf{B}_0^{\mathrm{H}})\mathrm{vec}(\mathbf{T})\} \\ & \mathrm{vec}^{\mathrm{H}}(\mathbf{T})(\mathbf{N}_1^* \otimes \mathbf{A}_1^{\mathrm{H}/2})(\mathbf{N}_1^{\mathrm{T}} \otimes \mathbf{A}_1^{1/2})\mathrm{vec}(\mathbf{T}) \\ & \quad \leq -c_1 - 2\mathcal{R}\{\mathrm{vec}^{\mathrm{H}}(\mathbf{B}_1^{\mathrm{H}})\mathrm{vec}(\mathbf{T})\} \\ & \mathrm{vec}^{\mathrm{H}}(\mathbf{T})(\mathbf{N}_2^* \otimes \mathbf{A}_2^{\mathrm{H}/2})(\mathbf{N}_2^{\mathrm{T}} \otimes \mathbf{A}_2^{1/2})\mathrm{vec}(\mathbf{T}) \\ & \quad \leq -c_2 - 2\mathcal{R}\{\mathrm{vec}^{\mathrm{H}}(\mathbf{B}_2^{\mathrm{H}})\mathrm{vec}(\mathbf{T})\}.\end{aligned} \tag{17}$$

Since c_0 does not affect the optimization problem, it has been neglected in (17).

With the Schur complement lemma [45], the optimization problem (17) can be further reformulated as the following semidefinite programming (SDP) problem [41]:

$$\begin{aligned}\min_{\mathbf{T},t} \quad & t \\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{I} & (\mathbf{N}_0^{\mathrm{T}} \otimes \mathbf{A}_0^{1/2})\mathrm{vec}(\mathbf{T}) \\ ((\mathbf{N}_0^{\mathrm{T}} \otimes \mathbf{A}_0^{1/2})\mathrm{vec}(\mathbf{T}))^{\mathrm{H}} & -2\mathcal{R}\{\mathrm{vec}^{\mathrm{H}}(\mathbf{B}_0)\mathrm{vec}(\mathbf{T})\} + t \end{bmatrix} \succcurlyeq 0, \\ & \begin{bmatrix} \mathbf{I} & (\mathbf{N}_l^{\mathrm{T}} \otimes \mathbf{A}_l^{1/2})\mathrm{vec}(\mathbf{T}) \\ ((\mathbf{N}_l^{\mathrm{T}} \otimes \mathbf{A}_l^{1/2})\mathrm{vec}(\mathbf{T}))^{\mathrm{H}} & -2\mathcal{R}\{\mathrm{vec}^{\mathrm{H}}(\mathbf{B}_l)\mathrm{vec}(\mathbf{T})\} - c_l \end{bmatrix} \succcurlyeq 0, \\ & l = 1,2.\end{aligned} \tag{18}$$

The precoder at the BS can then be designed by solving this SDP problem using standard numerical algorithms such as interior-point polynomial algorithms [39,41].

Please cite this article as: C. Xing, et al., Cooperative beamforming for dual-hop amplify-and-forward multi-antenna relaying cellular networks, Signal Processing (2012), http://dx.doi.org/10.1016/j.sigpro.2012.04.017

2.3. Summary and initialization

In summary, the downlink beamforming matrices are computed iteratively. Since in each iteration, the MSE monotonically decreases, the iterative algorithm is guaranteed to converge to at least a local optimum. For initialization, identity matrices can be chosen as initial values due to its simplicity and better performance compared to randomly generated initial matrices [30,31,36]. Alternatively, we can use a suboptimal design as initialization. Specifically, the downlink dual-hop AF MIMO relay cellular network is regarded as a combination of conventional point-to-point MIMO system in the first hop and multiuser MIMO downlink system in the second hop. The beamformers at two hops are designed independently. In other words, for the first hop, the linear minimum mean-square-error (LMMSE) precoder $\mathbf{T}$ at BS and equalizer $\mathbf{W}_1$ at relay station can be jointly designed using the point-to-point water-filling solution given in [29]. For the second hop, the precoder $\mathbf{W}_2$ at relay station and equalizer $\mathbf{G}$ at mobile terminals can be designed using the beamforming algorithm for multiuser MIMO systems proposed in [20]. Based on the results of $\mathbf{W}_1$ and $\mathbf{W}_2$, the initial value of the forwarding matrix at relay station is chosen as to $\mathbf{W}=\mathbf{W}_1\mathbf{W}_2$. We refer this suboptimal algorithm as 'separate LMMSE transceiver design'. It will be shown in Simulation section that the convergence speed using the second initialization is better than that of the first one. Finally, the iterative design procedure is formally given by

Algorithm 1. With initial $\mathbf{G}^0$, $\mathbf{W}^0$ and $\mathbf{T}^0$, the algorithm proceeds iteratively and in each iteration:

(1) $\mathbf{G}$ is updated using (9);
(2) $\mathbf{W}$ is updated using (13);
(3) $\mathbf{T}$ is updated by solving (18).

The algorithm stops when $\|\mathrm{MSE}_D^l-\mathrm{MSE}_D^{l+1}\|\le\mathcal{T}_D$, where MSE_D^l is the total MSE in the lth iteration and $\mathcal{T}_D$ is a threshold value. Notice that in each step of updating one matrix, the other two matrices are kept fixed.

3. Uplink beamforming design

3.1. System model and analogy with downlink design

In this section we will focus on beamforming matrices design for uplink, as shown in Fig. 2b. In uplink, there are L_k data streams to be transmitted from the kth mobile terminal to the BS, and the signal from the kth mobile terminal is denoted as $\mathbf{s}_k$. Without loss of generality, it is assumed that the transmitted data streams are independent: $\mathbb{E}\{\mathbf{s}_k\mathbf{s}_j^{\mathrm{H}}\}=\mathbf{0}_{L_k,L_j}$ when $k\neq j$ and $\mathbb{E}\{\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}\}=\mathbf{I}_{L_k}$. At the kth mobile terminal, the transmit signal $\mathbf{s}_k$ is multiplied by a precoder matrix $\mathbf{P}_k$ under a power constraint $\mathrm{Tr}(\mathbf{P}_k\mathbf{P}_k^{\mathrm{H}})\le P_{s,k}$, where $P_{s,k}$ is the maximum transmit power at the kth mobile terminal. The received signal $\mathbf{x}$ at the relay station is the superposition of signals from different terminals through different channels and is given by

$$\mathbf{x}=\mathbf{H}_{MR}\mathbf{P}\mathbf{s}+\mathbf{n}, \tag{19}$$

where $\mathbf{H}_{MR}\triangleq[\mathbf{H}_{MR,1}\ \cdots\ \mathbf{H}_{MR,K}]$, $\mathbf{P}\triangleq\mathrm{diag}\{[\mathbf{P}_1,\ldots,\mathbf{P}_K]\}$, $\mathbf{s}\triangleq[\mathbf{s}_1^{\mathrm{T}}\ \cdots\ \mathbf{s}_K^{\mathrm{T}}]^{\mathrm{T}}$, with $\mathbf{H}_{MR,k}$ being the $N_R\times N_{M,k}$ channel matrix between the kth mobile terminal and relay station, and $\mathbf{n}$ is the additive Gaussian noise at the relay station with zero mean and covariance matrix $\mathbf{R}_{\mathbf{n}}$. Since the data transmitted from different mobile terminals are independent, the correlation matrix of $\mathbf{x}$ equals to

$$\mathbf{R}_{\mathbf{x}}=\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}+\mathbf{R}_{\mathbf{n}}. \tag{20}$$

At the relay station, the received signal $\mathbf{x}$ is multiplied with a linear forwarding matrix $\mathbf{F}$, with a power constraint $\mathrm{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{x}}\mathbf{F}^{\mathrm{H}})\le P_r$, where P_r is the maximum transmit power at the relay station. Finally, the received signal at the BS is

$$\mathbf{y}=\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P}\mathbf{s}+\mathbf{H}_{RB}\mathbf{F}\mathbf{n}+\xi, \tag{21}$$

where $\mathbf{H}_{RB}$ is the $N_B\times N_R$ channel matrix between the relay station and BS, and ξ is the additive zero mean Gaussian noise with covariance $\mathbf{R}_{\xi}$.

When a linear equalizer $\mathbf{B}$ is adopted at the BS, the total MSE of the detected data is

$$\begin{aligned}\mathrm{MSE}_U(\mathbf{B},\mathbf{F},\mathbf{P})&=\mathbb{E}\{\|\mathbf{B}\mathbf{y}-\mathbf{s}\|^2\}\\&=\mathrm{Tr}(\mathbf{B}(\mathbf{H}_{RB}\mathbf{F}\mathbf{R}_{\mathbf{x}}\mathbf{F}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}}+\mathbf{R}_{\xi})\mathbf{B}^{\mathrm{H}})-\mathrm{Tr}(\mathbf{B}\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P})\\&\quad-\mathrm{Tr}((\mathbf{B}\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P})^{\mathrm{H}})+\mathrm{Tr}(\mathbf{I}_L),\end{aligned} \tag{22}$$

where $L=\sum_{k=1}^{K}L_k$ is the total number of data streams. Finally, the optimization problem for beamforming matrices design in the uplink case is formulated as

$$\begin{aligned}\min_{\mathbf{B},\mathbf{F},\mathbf{P}}\quad&\mathrm{MSE}_U(\mathbf{B},\mathbf{F},\mathbf{P})\\ \text{s.t.}\quad&\mathrm{Tr}(\mathbf{P}_k\mathbf{P}_k^{\mathrm{H}})\le P_{s,k},\quad k=1,\ldots,K,\\&\mathrm{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{x}}\mathbf{F}^{\mathrm{H}})\le P_r,\\&\mathbf{P}=\mathrm{diag}\{[\mathbf{P}_1,\ldots,\mathbf{P}_K]\}.\end{aligned} \tag{23}$$

Comparing (23) with the downlink problem (7), it can be seen that the two problems are in the same form, except that (i) there are individual constraints on $\mathbf{P}_k$ in (23) instead of a sum constraint on the corresponding $\mathbf{T}_k$ in (7) and (ii) the diagonal structure constraint is on precoder instead of equalizer. However we can still employ the iterative algorithm developed in the previous section for this uplink beamforming design problem. More specifically, for equalizer design of $\mathbf{B}$, the problem is an unconstrained convex optimization problem and the optimal solution can be computed from the derivative of the objective function. For forwarding matrix design of $\mathbf{F}$, the problem is a convex quadratic optimization problem with only one constraint. In this case, the optimal solution can be solved based on KKT conditions. Finally, for precoder design of $\mathbf{P}$, the problem is a convex quadratic optimization with multiple constraints, which can also be transformed into a standard SDP problem. Notice that a SDP problem can handle any number of linear matrix inequality constraints and the diagonal structure of $\mathbf{P}$ does not affect the SDP problem.

Although the optimization problem (23) can be solved using the proposed iterative algorithm alternating among the three variables $\mathbf{B}$, $\mathbf{F}$ and $\mathbf{P}$, this solution provides little insight into the nature of the problem. Furthermore, in each iteration, three matrices need to be computed and bisection search is required for the computation of the forwarding matrix at the relay, resulting in high computational complexity. Below we will propose a beamforming design algorithm with lower complexity for the fully loaded or overloaded uplink. The solution is also found to be insightful and includes several existing algorithms for conventional AF MIMO relay or multiuser MIMO as special cases.

3.2. Uplink beamforming design

First, we reduce the number of variables of the optimization problem. Noticing that there is no constraint on $\mathbf{B}$, the optimal $\mathbf{B}$ satisfies $\partial \mathrm{MSE}_U(\mathbf{B},\mathbf{F},\mathbf{P})/\partial \mathbf{B}^* = \mathbf{0}_{L,N_B}$, and the optimal equalizer at the BS can be written as a function of forwarding matrix and precoder matrices. Therefore

$$\mathbf{B} = (\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P})^{\mathrm{H}}(\mathbf{H}_{RB}\mathbf{F}\mathbf{R}_{\mathbf{x}}\mathbf{F}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}} + \mathbf{R}_{\xi})^{-1}. \tag{24}$$

Substituting this result into (22), the uplink MSE is simplified as

$$\mathrm{MSE}_U(\mathbf{F},\mathbf{P}) = \mathrm{Tr}(\mathbf{I}_L) - \mathrm{Tr}((\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P})^{\mathrm{H}}(\mathbf{H}_{RB}\mathbf{F}\mathbf{R}_{\mathbf{x}}\mathbf{F}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}} + \mathbf{R}_{\xi})^{-1}(\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P})). \tag{25}$$

Based on the definition of $\mathbf{R}_{\mathbf{x}} = \mathbf{H}_{MR}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}^{\mathrm{H}} + \mathbf{R}_{\mathbf{n}}$, it can be expressed as

$$\mathbf{R}_{\mathbf{x}} = \mathbf{R}_{\mathbf{n}}^{1/2}\underbrace{(\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}^{\mathrm{H}}\mathbf{R}_{\mathbf{n}}^{-1/2} + \mathbf{I})}_{\triangleq \boldsymbol{\Xi}}\mathbf{R}_{\mathbf{n}}^{1/2}. \tag{26}$$

Now introducing $\tilde{\mathbf{F}} = \mathbf{F}\mathbf{R}_{\mathbf{n}}^{1/2}\boldsymbol{\Xi}^{1/2}$, the MSE (25) becomes

$$\overline{\mathrm{MSE}}_U(\tilde{\mathbf{F}},\mathbf{P}) = \mathrm{Tr}(\mathbf{I}_L) - \mathrm{Tr}((\mathbf{H}_{RB}\tilde{\mathbf{F}}\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P})^{\mathrm{H}} \times (\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}} + \mathbf{R}_{\xi})^{-1}(\mathbf{H}_{RB}\tilde{\mathbf{F}}\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P})). \tag{27}$$

Thus the uplink beamforming design optimization problem (23) is rewritten as

$$\begin{aligned} \min_{\tilde{\mathbf{F}},\mathbf{P}} \quad & \overline{\mathrm{MSE}}_U(\tilde{\mathbf{F}},\mathbf{P}) \\ \text{s.t.} \quad & \mathrm{Tr}(\mathbf{P}_k\mathbf{P}_k^{\mathrm{H}}) \le P_{s,k}, \quad k = 1,\ldots,K, \\ & \mathrm{Tr}(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^{\mathrm{H}}) \le P_r, \\ & \mathbf{P} = \mathrm{diag}\{[\mathbf{P}_1,\ldots,\mathbf{P}_K]\}. \end{aligned} \tag{28}$$

Notice that $\tilde{\mathbf{F}}$ is a function of $\mathbf{F}$ and $\mathbf{P}$ as $\tilde{\mathbf{F}} = \mathbf{F}\mathbf{R}_{\mathbf{n}}^{1/2}\boldsymbol{\Xi}^{1/2}$. For the optimization problem (28), the variables in the objective function are only $\tilde{\mathbf{F}}$ and $\mathbf{P}$. Furthermore, for any given $\tilde{\mathbf{F}}$ and $\mathbf{P}$, we can directly reconstruct a unique $\mathbf{F}$ satisfying the constraints. Therefore, the constraint between $\tilde{\mathbf{F}}$ and $\mathbf{F}$ and $\mathbf{P}$, i.e., $\tilde{\mathbf{F}} = \mathbf{F}\mathbf{R}_{\mathbf{n}}^{1/2}\boldsymbol{\Xi}^{1/2}$, can be ignored in the optimization problem (28).

Unfortunately, the optimization problem (28) is still nonconvex for $\tilde{\mathbf{F}}$ and $\mathbf{P}$, and thus there is no closed-form solution. However, notice that if either $\tilde{\mathbf{F}}$ or $\mathbf{P}$ is fixed, the optimization problem is convex with respect to the remaining variable. Therefore, an iterative algorithm which designs $\tilde{\mathbf{F}}$ and $\mathbf{P}$ alternatively is proposed as follows.

(1) *Design* $\tilde{\mathbf{F}}$ *when* $\mathbf{P}$ *is fixed*: From (27), it is noticed that $\tilde{\mathbf{F}}$ appears both inside and outside of the inverse operation. In order to simplify the objective function, we use the following variant of matrix inversion lemma:

$$\mathbf{C}^{\mathrm{H}}(\mathbf{C}\mathbf{C}^{\mathrm{H}} + \mathbf{D})^{-1}\mathbf{C} = \mathbf{I} - (\mathbf{C}^{\mathrm{H}}\mathbf{D}^{-1}\mathbf{C} + \mathbf{I})^{-1}. \tag{29}$$

Taking $\mathbf{C} = \mathbf{H}_{RB}\tilde{\mathbf{F}}$ and $\mathbf{D} = \mathbf{R}_{\xi}$, the MSE (27) can be reformulated as [40]

$$\begin{aligned} \mathrm{MSE}_U(\tilde{\mathbf{F}},\mathbf{P}) = {} & \mathrm{Tr}((\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P})(\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}_r}^{-1/2}\mathbf{H}_{MR}\mathbf{P})^{\mathrm{H}} \\ & \times(\tilde{\mathbf{F}}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}}\mathbf{R}_{\xi}^{-1}\mathbf{H}_{RB}\tilde{\mathbf{F}} + \mathbf{I})^{-1}) \\ & + \mathrm{Tr}((\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}^{\mathrm{H}}\mathbf{R}_{\mathbf{n}}^{-1}\mathbf{H}_{MR}\mathbf{P} + \mathbf{I})^{-1}). \end{aligned} \tag{30}$$

Now, $\tilde{\mathbf{F}}$ only appears inside the matrix inverse. If $\mathbf{P}$ is fixed, the last term of (30) is independent of $\tilde{\mathbf{F}}$, and the optimization problem (28) becomes

$$\begin{aligned} \min_{\tilde{\mathbf{F}}} \quad & \mathrm{Tr}(\underbrace{(\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P})(\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P})^{\mathrm{H}}}_{\triangleq \boldsymbol{\Theta}} \\ & (\tilde{\mathbf{F}}^{\mathrm{H}}\underbrace{\mathbf{H}_{RB}^{\mathrm{H}}\mathbf{R}_{\xi}^{-1}\mathbf{H}_{RB}}_{\triangleq \mathbf{M}}\tilde{\mathbf{F}} + \mathbf{I})^{-1}) \\ \text{s.t.} \quad & \mathrm{Tr}(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^{\mathrm{H}}) \le P_r. \end{aligned} \tag{31}$$

Based on eigen-decompositions of $\boldsymbol{\Theta} = \mathbf{U}_{\boldsymbol{\Theta}}\boldsymbol{\Lambda}_{\boldsymbol{\Theta}}\mathbf{U}_{\boldsymbol{\Theta}}^{\mathrm{H}}$ and $\mathbf{M} = \mathbf{U}_{\mathbf{M}}\boldsymbol{\Lambda}_{\mathbf{M}}\mathbf{U}_{\mathbf{M}}^{\mathrm{H}}$, and defining

$$\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}} \triangleq \mathbf{U}_{\mathbf{M}}^{\mathrm{H}}\tilde{\mathbf{F}}\mathbf{U}_{\boldsymbol{\Theta}}, \tag{32}$$

the optimization problem (31) can be simplified as

$$\begin{aligned} \min_{\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}}} \quad & \mathrm{Tr}(\boldsymbol{\Lambda}_{\boldsymbol{\Theta}}(\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}}^{\mathrm{H}}\boldsymbol{\Lambda}_{\mathbf{M}}\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}} + \mathbf{I})^{-1}) \\ \text{s.t.} \quad & \mathrm{Tr}(\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}}\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}}^{\mathrm{H}}) \le P_r. \end{aligned} \tag{33}$$

Without loss of generality, the diagonal elements of $\boldsymbol{\Lambda}_{\boldsymbol{\Theta}}$ and $\boldsymbol{\Lambda}_{\mathbf{M}}$ are assumed to be arranged in decreasing order. The closed-form solution of (33) can be shown to be [40]

$$\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}} = \begin{bmatrix} \left[\left(\frac{1}{\sqrt{\mu_f}}\tilde{\boldsymbol{\Lambda}}_{\mathbf{M}}^{-1/2}\tilde{\boldsymbol{\Lambda}}_{\boldsymbol{\Theta}}^{1/2} - \tilde{\boldsymbol{\Lambda}}_{\mathbf{M}}^{-1}\right)^{+}\right]^{1/2} & \mathbf{0}_{L,N_R-L} \\ \mathbf{0}_{N_R-L,L} & \mathbf{0}_{N_R-L,N_R-L} \end{bmatrix}, \tag{34}$$

where $\tilde{\boldsymbol{\Lambda}}_{\boldsymbol{\Theta}}$ and $\tilde{\boldsymbol{\Lambda}}_{\mathbf{M}}$ are the $L \times L$ principal submatrices of $\boldsymbol{\Lambda}_{\boldsymbol{\Theta}}$ and $\boldsymbol{\Lambda}_{\mathbf{M}}$, respectively. The scalar μ_f is the Lagrange multiplier which makes $\mathrm{Tr}(\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}}\boldsymbol{\Lambda}_{\tilde{\mathbf{F}}}^{\mathrm{H}}) = P_r$ hold. Based on (32) and (34), the optimal $\tilde{\mathbf{F}}$ can be recovered as

$$\tilde{\mathbf{F}} = \mathbf{U}_{\mathbf{M},L}\left[\left(\frac{1}{\sqrt{\mu_f}}\tilde{\boldsymbol{\Lambda}}_{\mathbf{M}}^{-1/2}\tilde{\boldsymbol{\Lambda}}_{\boldsymbol{\Theta}}^{1/2} - \tilde{\boldsymbol{\Lambda}}_{\mathbf{M}}^{-1}\right)^{+}\right]^{1/2}\mathbf{U}_{\boldsymbol{\Theta},L}^{\mathrm{H}}, \tag{35}$$

where $\mathbf{U}_{\mathbf{M},L}$ and $\mathbf{U}_{\boldsymbol{\Theta},L}$ are the first L columns of $\mathbf{U}_{\mathbf{M}}$ and $\mathbf{U}_{\boldsymbol{\Theta}}$, respectively. Finally, the optimal $\mathbf{F}$ is given by $\mathbf{F} = \tilde{\mathbf{F}}\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}}^{-1/2}$.

(2) *Design* $\mathbf{P}$ *when* $\tilde{\mathbf{F}}$ *is fixed*: Since $\boldsymbol{\Xi}$ in (27) depends on $\mathbf{P}$, the MSE expression in (27) is a complicated function of $\mathbf{P}$, direct optimization of $\mathbf{P}$ seems intractable. However, based on the property of trace operator $\mathrm{Tr}(\mathbf{DC}) = \mathrm{Tr}(\mathbf{CD})$, the total MSE (27) can be reformulated as [44]

$$\begin{aligned} \overline{\mathrm{MSE}}_U(\tilde{\mathbf{F}},\mathbf{P}) = {} & \mathrm{Tr}(\mathbf{I}_L) - \mathrm{Tr}((\mathbf{H}_{RB}\tilde{\mathbf{F}})^{\mathrm{H}}(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}} + \mathbf{R}_{\xi})^{-1} \\ & \times\mathbf{H}_{RB}\tilde{\mathbf{F}})(\boldsymbol{\Xi}^{-1/2}\underbrace{\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}^{\mathrm{H}}\mathbf{R}_{\mathbf{n}}^{-1/2}}_{=\boldsymbol{\Xi}-\mathbf{I}}\boldsymbol{\Xi}^{-1/2})) \\ = {} & \mathrm{Tr}(\mathbf{I}_L) - \mathrm{Tr}((\mathbf{H}_{RB}\tilde{\mathbf{F}})^{\mathrm{H}}(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}} \\ & + \mathbf{R}_{\xi})^{-1}\mathbf{H}_{RB}\tilde{\mathbf{F}})(\mathbf{I}_{N_R} - \boldsymbol{\Xi}^{-1})). \end{aligned} \tag{36}$$

Substituting the definition of $\boldsymbol{\Xi}$ into (36), the MSE can be further rewritten as

$$\begin{aligned}\overline{\mathrm{MSE}}_U(\tilde{\mathbf{F}},\mathbf{P}) = &\mathrm{Tr}(\underbrace{(\mathbf{H}_{RB}\tilde{\mathbf{F}})^{\mathrm{H}}(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}}+\mathbf{R}_{\xi})^{-1}(\mathbf{H}_{RB}\tilde{\mathbf{F}})}_{\triangleq \boldsymbol{\Pi}}\\ &\times(\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}^{\mathrm{H}}\mathbf{R}_{\mathbf{n}}^{-1/2}+\mathbf{I}_{N_R})^{-1})\\ &+\mathrm{Tr}(\mathbf{I}_L)-\mathrm{Tr}((\mathbf{H}_{RB}\tilde{\mathbf{F}})^{\mathrm{H}}(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}}+\mathbf{R}_{\xi})^{-1}(\mathbf{H}_{RB}\tilde{\mathbf{F}})),\end{aligned} \tag{37}$$

where $\mathbf{P}$ only appears inside of the inverse operation. As the last two terms of (37) are independent of $\mathbf{P}$, the optimization problem for $\mathbf{P}$ is equivalent to

$$\begin{aligned}\min_{\mathbf{P}}\quad & \mathrm{Tr}(\boldsymbol{\Pi}(\mathbf{R}_{\mathbf{n}}^{-1/2}\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}^{\mathrm{H}}\mathbf{R}_{\mathbf{n}}^{-1/2}+\mathbf{I}_{N_R})^{-1})\\ \text{s.t.}\quad & \mathrm{Tr}(\mathbf{P}_k\mathbf{P}_k^{\mathrm{H}})\le P_{s,k},\quad k=1,\ldots,K,\\ & \mathbf{P}=\mathrm{diag}\{[\mathbf{P}_1,\ldots,\mathbf{P}_K]\}.\end{aligned} \tag{38}$$

With the definitions of $\mathbf{H}_{MR}$ and $\mathbf{P}$

$$\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_{MR}^{\mathrm{H}} = \sum_{k=1}^{K}\{\mathbf{H}_{MR,k}\underbrace{\mathbf{P}_k\mathbf{P}_k^{\mathrm{H}}}_{\triangleq\mathbf{Q}_k}\mathbf{H}_{MR,k}^{\mathrm{H}}\}. \tag{39}$$

For a fully or over loaded uplink, i.e., $N_{M,k}\le L_k$, putting (39) into (38) reduces the optimization problem as

$$\begin{aligned}\min_{\mathbf{Q}_k}\quad & \mathrm{Tr}\left(\boldsymbol{\Pi}(\mathbf{R}_{\mathbf{n}}^{-1/2}\sum_{k=1}^{K}\{\mathbf{H}_{MR,k}\mathbf{Q}_k\mathbf{H}_{MR,k}^{\mathrm{H}}\}\mathbf{R}_{\mathbf{n}}^{-1/2}+\mathbf{I}_{N_R})^{-1}\right)\\ \text{s.t.}\quad & \mathrm{Tr}(\mathbf{Q}_k)\le P_{s,k},\quad k=1,\ldots,K,\\ & \mathbf{Q}_k\succcurlyeq\mathbf{0}.\end{aligned} \tag{40}$$

Using the Schur-complement lemma [45], the optimization problem (40) can be further formulated as a standard SDP optimization problem [44]

$$\begin{aligned}\min_{\mathbf{X},\mathbf{Q}_k}\quad & \mathrm{Tr}(\mathbf{X})\\ \text{s.t.}\quad & \begin{bmatrix}\mathbf{X} & \boldsymbol{\Pi}^{1/2}\\ \boldsymbol{\Pi}^{1/2} & \mathbf{R}_{\mathbf{n}}^{-1/2}\sum_{k}\{\mathbf{H}_{MR,k}\mathbf{Q}_k\mathbf{H}_{MR,k}^{\mathrm{H}}\}\mathbf{R}_{\mathbf{n}}^{-1/2}+\mathbf{I}_{N_R}\end{bmatrix}\succcurlyeq\mathbf{0},\\ & \mathrm{Tr}(\mathbf{Q}_k)\le P_{s,k},\quad k=1,\ldots,K,\\ & \mathbf{Q}_k\succcurlyeq\mathbf{0}.\end{aligned} \tag{41}$$

It then can be efficiently solved using interior-point polynomial algorithms [39].

In summary, when $N_{M,k}\le L_k$, i.e., the uplink system is fully or over loaded, the beamforming design alternates between the design of $\tilde{\mathbf{F}}$ in (35) and $\mathbf{Q}_k$ in (41). The algorithm stops when $\|\mathrm{MSE}_U^I-\mathrm{MSE}_U^{I+1}\|\le\mathcal{T}_U$, where MSE_U^I is the total MSE in the Ith iteration and $\mathcal{T}_U$ is a threshold value. After convergence, $\mathbf{P}_k=\mathbf{Q}_k^{1/2}$, $\mathbf{F}=\tilde{\mathbf{F}}\boldsymbol{\Xi}^{-1/2}\mathbf{R}_{\mathbf{n}}^{-1/2}$ and $\mathbf{B}=(\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P})^{\mathrm{H}}(\mathbf{H}_{RB}\mathbf{F}\mathbf{R}_{\mathbf{x}}\mathbf{F}^{\mathrm{H}}\mathbf{H}_{RB}^{\mathrm{H}}+\mathbf{R}_{\xi})^{-1}$. We refer the algorithm in this section as Algorithm 2.

Remark 1. In case $N_{M,k}>L_k$, there is an additional constraint $\mathrm{Rank}\{\mathbf{Q}_k\}\le N_k$ in (40). In this case, as rank constraints are nonconvex, transition from (40) to (41) involves a relaxation on the rank constraint. Then the objective function of (41) is a lower bound of that of (40). However, this problem seems to be common to all multiuser MIMO uplink beamforming [33,35].Notice that when $N_{M,k}\le L_k$ (fully loaded or overloaded MIMO systems [42,43]), there is no relaxation involved.

Remark 2. The proposed iterative algorithm in this section promises lower complexity compared with that in the previous section, as there are only two variables $\tilde{\mathbf{F}}$ and $\mathbf{P}$ to be solved instead of three. Furthermore, $\tilde{\mathbf{F}}$ has a closed-form solution without the need of bisection search for the Lagrange multiplier.

3.3. Special cases

Notice that (35) has a more general form than the water-filling solution in traditional point-to-point MIMO systems. On the other hand, (41) is a SDP problem frequently encountered in multiuser MIMO systems. In particular, they include the following existing algorithms as special cases:

- If $\mathbf{H}_{RB}=\mathbf{I}_L$ and $\mathbf{R}_{\xi}=\mathbf{0}_{L,L}$, we have $\boldsymbol{\Pi}=\mathbf{I}_L$ in (40), and the SDP optimization problem (41) reduces to that of the uplink multiuser MIMO systems [33,35]. Therefore, they have the same solution.
- Substituting $K=1$ and $\mathbf{P}=\mathbf{I}_{L_1}$ into (35), the optimal solution reduces to that proposed for joint design of relay forwarding matrix and destination equalizer in AF MIMO relay systems without source precoder in [3].
- When there is only one mobile terminal ($K=1$), the optimization problem (38) is in the same form as (31). Defining $\mathbf{H}_{MR}^{\mathrm{H}}\mathbf{R}_{\mathbf{n}}^{-1}\mathbf{H}_{MR}=\mathbf{U}_{MR}\boldsymbol{\Lambda}_{MR}\mathbf{U}_{MR}^{\mathrm{H}}$, and $\boldsymbol{\Pi}=\mathbf{U}_{\boldsymbol{\Pi}}\boldsymbol{\Lambda}_{\boldsymbol{\Pi}}\mathbf{U}_{\boldsymbol{\Pi}}^{\mathrm{H}}$, a closed-form solution can be derived using the same procedure as for $\tilde{\mathbf{F}}$, and we have
$$\mathbf{P}=\mathbf{U}_{MR,L}\left[\left(\frac{1}{\sqrt{\mu_p}}\tilde{\boldsymbol{\Lambda}}_{MR}^{-1/2}\tilde{\boldsymbol{\Lambda}}_{\boldsymbol{\Pi}}^{1/2}-\tilde{\boldsymbol{\Lambda}}_{MR}^{-1}\right)^{+}\right]^{1/2}, \tag{42}$$
where $\tilde{\boldsymbol{\Lambda}}_{MR}$ and $\tilde{\boldsymbol{\Lambda}}_{\boldsymbol{\Pi}}$ are the $L\times L$ principal submatrices of $\boldsymbol{\Lambda}_{MR}$ and $\boldsymbol{\Lambda}_{\boldsymbol{\Pi}}$, respectively, and the matrix $\mathbf{U}_{MR,L}$ is the first L columns of $\mathbf{U}_{MR}$. The scalar μ_p is the Lagrange multiplier which makes $\mathrm{Tr}(\mathbf{P}\mathbf{P}^{\mathrm{H}})=P_{s,1}$ hold. In this case, the solution given by (42) corresponds to the source precoder design for AF MIMO relay systems with single user [5].
- Furthermore, substituting $\mathbf{H}_{RB}=\mathbf{I}_L$ and $\mathbf{R}_{\xi}=\mathbf{0}_{L,L}$ into (42), the proposed solution becomes the closed-form solution for LMMSE transceiver design in point-to-point MIMO systems [29].

4. Simulation results and discussions

In this section, we investigate the performance of the proposed algorithms for downlink and uplink. In the simulations, there is one BS, one relay station and two mobile terminals. For each mobile terminal, two independent data streams will be transmitted in the uplink (or received in the downlink) simultaneously. For each data stream, 10 000 independent QPSK symbols are transmitted. The elements of MIMO channels between BS and relay station and between relay station and mobile terminals are generated as independent complex Gaussian random variables with zero mean and unit variance. Each point in the following figures is an average of 500 independent channel realizations. In order to solve SDP problems, the widely used optimization Matlab toolbox

Please cite this article as: C. Xing, et al., Cooperative beamforming for dual-hop amplify-and-forward multi-antenna relaying cellular networks, Signal Processing (2012), http://dx.doi.org/10.1016/j.sigpro.2012.04.017

CVX is adopted [48]. The thresholds for terminating the iterative algorithms are set at $\mathcal{T}_D = \mathcal{T}_U = 0.0001$.

The downlink performance is first investigated. In the downlink, the noise covariance matrices at relay station and mobile terminals are $\mathbf{R}_{\boldsymbol{\eta}} = \sigma_\eta^2 \mathbf{I}_{N_R}$ and $\mathbf{R}_{\mathbf{v}_1} = \mathbf{R}_{\mathbf{v}_2} = \sigma_v^2 \mathbf{I}_{N_M}$, respectively. We define the first hop SNR at the relay station as P_s/σ_η^2, and the second hop SNR at mobile terminals as P_r/σ_v^2. Fig. 3 shows the convergence behavior of the proposed Algorithm 1 for downlink with different second hop SNR at mobile terminals when $N_B=4$, $N_R=4$, $N_{M,k}=2$. Both initializations with identity matrices and the separate LMMSE design are simulated. Notice that in each step of obtaining one matrix in the iterative algorithm, the other two matrices are kept fixed. It can be seen that the proposed algorithm converges quickly, within 20 iterations. Furthermore, the convergence speed with separate LMMSE design as initialization is faster than that with identity matrices. It can also be seen that the two initializations result in the same MSE after convergence.

Fig. 4 compares the total data MSEs of the proposed Algorithm 1 and several suboptimal algorithms versus the first hop SNR P_s/σ_η^2. The second hop SNR at mobile terminals is fixed to be 20 dB. The number of antennas is set as $N_B=4$, $N_R=4$ and $N_{M,k}=2$. The suboptimal algorithms under consideration are

- Equalization at mobile terminals only, in which the precoder $\mathbf{T}$ at BS and forwarding matrix $\mathbf{W}$ at relay are proportional to identity matrices. At mobile terminals, LMMSE equalizer for the combined first hop and second hop channel is adopted to recover the signal [3].
- Equalization at both relay and mobile terminals, in which the first hop channel is equalized at the relay and then the second hop channel is equalized at mobile terminals, both following LMMSE criterion.
- Separate LMMSE design for two hops, which is the algorithm proposed for initialization of Algorithm 1.

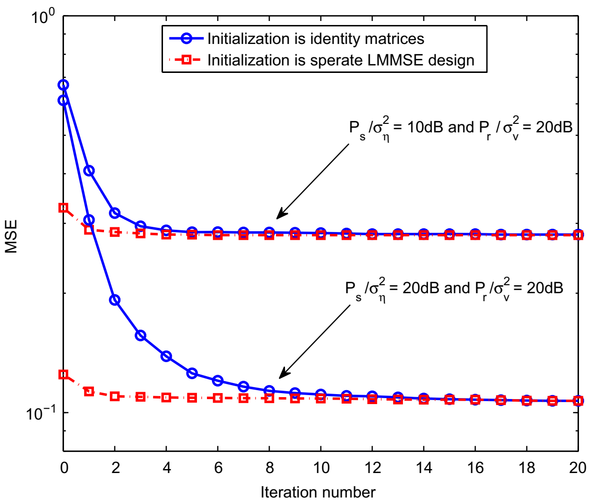

Fig. 3. The convergence behavior of the proposed Algorithm 1 when $N_B=4$, $N_R=4$ and $N_{M,k}=2$ with two users.

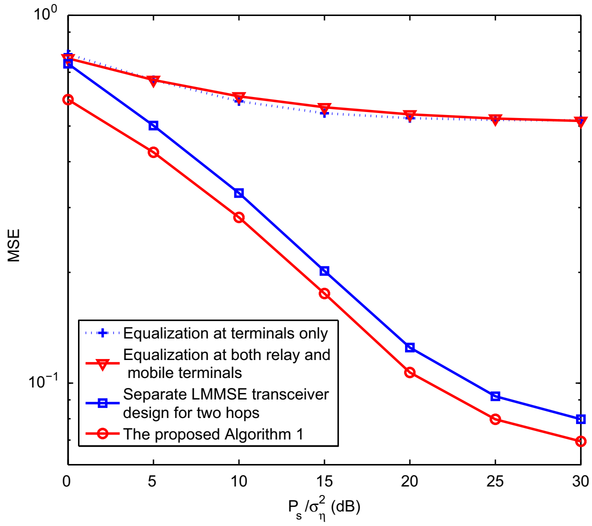

Fig. 4. Total MSEs of detected data of the proposed Algorithm 1 and suboptimal algorithms, when $N_B=4$, $N_R=4$, $N_{M,k}=2$ and $P_r/\sigma_v^2 = 20$ dB.

From Fig. 4, it can be seen that as there is no precoder design at BS for the first two suboptimal algorithms, the data streams at different terminals cannot be efficiently separated by linear equalizers, resulting in poor performances. The separate LMMSE transceiver design has a much better performance. On the other hand, the proposed Algorithm 1 has the best performance among the four algorithms. The gap between the MSEs of the separate LMMSE design and that of Algorithm 1 is the performance gain obtained by additional iterations. In other words, Algorithm 1 offers better performance with higher complexity, comparing with the separate LMMSE design. Specifically, there exists certain tradeoff between performance and complexity in these two algorithms.

As the proposed Algorithm 1 involves a computational expensive SDP for the precoder $\mathbf{T}$ design, it is of great interest to investigate how much degradation would result from skipping the precoder design. Fig. 5 compares the total data MSEs of the proposed Algorithm 1 and the same algorithm but fixing the precoder $\mathbf{T} \propto \mathbf{I}$. The second hop SNR at mobile terminals P_r/σ_v^2 is fixed to be 20 dB. From Fig. 5, it can be seen that a properly designed precoder significantly improves the system performance when the first hop SNR is high. Without the precoder, the data MSEs exhibit error floors at much lower P_s/σ_η^2. On the other hand, we can also see that increasing the number of antennas at the relay station greatly improves the system performance, as it simultaneously increases the diversity gain of the two hops.

Now, let us turn to the results in the uplink. In uplink case, the noise covariance matrices at relay station and BS are $\mathbf{R}_{\mathbf{n}} = \sigma_n^2 \mathbf{I}_{N_R}$ and $\mathbf{R}_\xi = \sigma_\xi^2 \mathbf{I}_{N_B}$, respectively. We define the first hop SNR at the relay station as P_s/σ_n^2, where $P_s = \sum_{k=1}^K P_{s,k}$. The second hop SNR at the BS is defined as P_r/σ_ξ^2.

Fig. 6 shows the convergence behavior of the proposed Algorithm 2 for uplink when $N_B=4$, $N_R=4$ and $N_{M,k}=2$. Notice that in this case, at each mobile terminal the

Please cite this article as: C. Xing, et al., Cooperative beamforming for dual-hop amplify-and-forward multi-antenna relaying cellular networks, Signal Processing (2012), http://dx.doi.org/10.1016/j.sigpro.2012.04.017

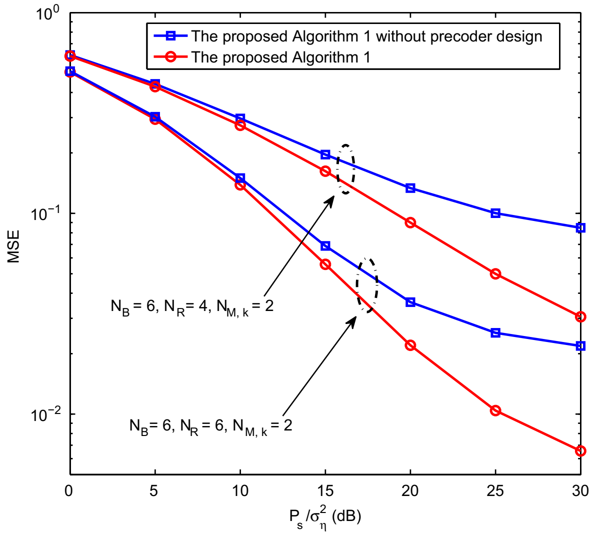

Fig. 5. Total MSEs of detected data of the proposed Algorithm 1 with and without precoder design.

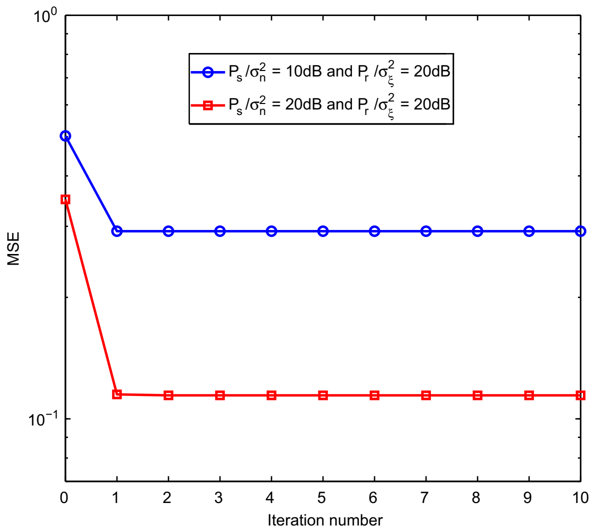

Fig. 6. The convergence behavior of Algorithm 2 for uplink when $N_B=4$, $N_R=4$ and $N_{M,k}=2$.

number of antennas equals to that of the data streams, and Algorithm 2 involves no relaxation. Identity matrices are chosen for initialization. It can be seen that Algorithm 2 converges very fast, indicating its superior performance.

Fig. 7 shows the total data MSEs of the proposed Algorithm 2 and suboptimal algorithms, when $N_B=4$, $N_R=4$, $N_{M,k}=2$ and the SNR at relay station P_s/σ_n^2 is fixed to be 20 dB. The suboptimal algorithms are similar to those for the downlink. In particular, we consider

- Equalization of the equivalent two-hop channel is applied only at the BS.
- Equalization is applied at relay station for the mobile-to-relay channel, and also at BS for the relay-to-BS channel.

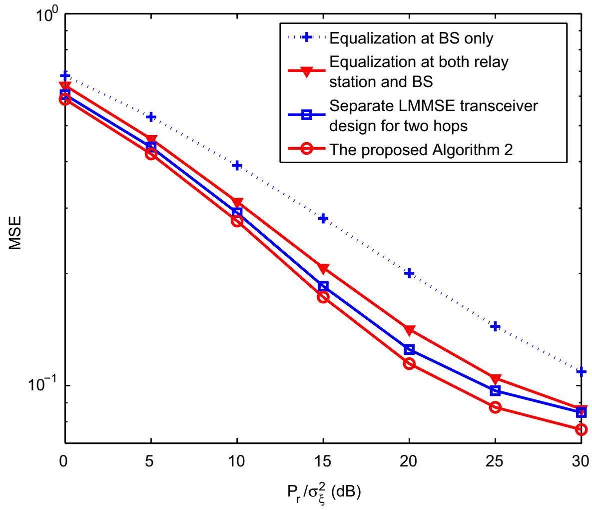

Fig. 7. Total MSEs of detected data of Algorithm 2 and suboptimal algorithms, when $N_B=4$, $N_R=4$, $N_{M,k}=2$ and $P_s/\sigma_n^2=20$ dB.

- Separate LMMSE design. The first hop is considered as a traditional multiuser MIMO uplink system, and the beamforming matrices are designed using the algorithms in [31,33]. The second hop is considered as a point-to-point MIMO system, and the beamforming matrices are designed using the result in [29].

From Fig. 7, it can be seen that the performance of the proposed Algorithm 2 is better than that of the other suboptimal algorithms. However, as the signals from different terminals are cooperatively detected at BS, the gaps between the performance of the suboptimal algorithms from that of Algorithm 2 are much smaller compared to their counterparts in downlink.

When $L_k < N_{M,k}$ in the uplink, strictly speaking, Algorithm 2 involves a relaxation, and its performance is not guaranteed. However, a simple variation of Algorithm 1 can be used for beamforming design in this case. Fig. 8 shows the total data MSEs of Algorithm 1 for uplink and Algorithm 2 with rank relaxation, when $L_k=2$ and $N_{M,k}=4$. The SNR at BS is fixed at $P_r/\sigma_\xi^2=20$ dB. The joint design of relay forwarding matrix and destination equalizer in [3] is also shown for comparison. It can be viewed as a design without source precoders at mobile terminals. From Fig. 8, it can be seen that Algorithms 1 and 2, which involve the joint design of precoder, forwarding matrix and equalizer, perform better than the algorithm in [3]. This indicates the importance of source precoder design in AF relay cellular networks. Furthermore, although Algorithm 2 involves a relaxation, its performance is still satisfactory and is close to that of Algorithm 1. Finally, it can also be concluded that increasing the number of antennas at relay station can greatly improve the performance of uplink beamforming design for all algorithms.

5. Conclusions

In this paper, LMMSE beamforming design for amplify-and-forward MIMO relay cellular networks has been

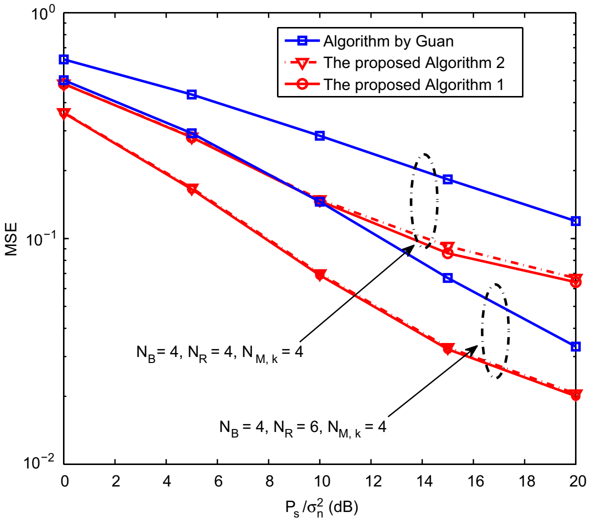

Fig. 8. Total MSEs of the detected data of the Algorithms 1 and 2 with relaxation and the algorithm proposed in [3].

investigated. Both uplink and downlink cases were considered. In the downlink, precoder at base station, forwarding matrix at relay station and equalizers at mobile terminals were jointly designed by an iterative algorithm. On the other hand, in the uplink case, we demonstrated that in general the beamforming design problem can be solved by an iterative algorithm with the same structure as in the downlink case. Furthermore, for the fully loaded or overloaded uplink systems, a novel beamforming design algorithm with lower complexity was derived and it includes several existing algorithms for conventional point-to-point or multiuser systems as special cases. Finally, simulation results have corroborated the performance advantage of the proposed algorithms over several suboptimal schemes.

Acknowledgment

This work was supported in part by the National Natural Science Foundation of China (NSFC) under Grant 61101130, the HKU Seed Funding Programme, Project No. 201111159122 and China-EU International Scientific and Technological Cooperation Program (0902).

References

[1] A. Scaglione, D.L. Goeckel, J.N. Laneman, Cooperative communications in mobile ad hoc networks, IEEE Signal Processing Magazine 23 (5) (2006) 18–29.
[2] J.N. Laneman, D.N.C. Tse, G.W. Wornell, Cooperative diversity in wireless networks: efficient protocols and outage behavior, IEEE Transactions on Information Theory 50 (12) (2004) 3062–3080.
[3] W. Guan, H. Luo, Joint MMSE transceiver design in non-regenerative MIMO relay systems, IEEE Communications Letters 12 (7) (2008) 517–519.
[4] X. Tang, Y. Hua, Optimal design of non-regenerative MIMO wireless relays, IEEE Transactions on Wireless Communications 6 (4) (2007) 1398–1407.
[5] Y. Rong, X. Tang, Y. Hua, A unified framework for optimizing linear non-regenerative multicarrier MIMO relay communication systems, IEEE Transactions on Signal Processing 57 (12) (2009) 4837–4851.
[6] S. Ma, C. Xing, Y. Fan, Y.-C. Wu, T.-S. Ng, H. Vincent Poor, Iterative transceiver design for MIMO AF relay networks with multiple sources, in: Proceedings of MILCOM 2010, USA, 2010 (Invited paper).
[7] Y. Rong, F. Gao, Optimal beamforming for non-regenerative MIMO relays with direct link, IEEE Communications Letters 13 (12) (2009) 926–928.
[8] Y. Jing, H. Jafarkhani, Network beamforming using relays with perfect channel information, IEEE Transactions on Information Theory 55 (2009) 2499–2517.
[9] N. Khajehnouri, A.H. Sayed, Distributed MMSE relay strategies for wireless sensor networks, IEEE Transactions on Signal Processing 55 (7) (2007) 3336–3348.
[10] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, Z.-Q. Luo, Distributed beamforming for relay networks based on second-order statistics of the channel state information, IEEE Transactions on Signal Processing 56 (9) (2008) 4306–4316.
[11] J. Choi, J. Ha, MMSE-based distributed beamforming in cooperative relay networks, IEEE Transactions on Communications 59 (5) (2011) 1346–1356.
[12] E. Gharavol, Y. Liang, K. Mouthaan, Robust linear beamforming for MIMO relay with imperfect channel state information, in: Proceedings of PIMRC 2010, Istanbul, Turkey, 2010.
[13] G. Li, Y. Wang, P. Zhang, Optimal linear MMSE beamforming for two way multi-antenna relay systems, IEEE Communications Letters 15 (5) (2011) 533–535.
[14] I. Hammerstrom, M. Kuhn, C. Esli, J. Zhao, A. Wittneben, G. Bauch, MIMO two-way relaying with transmit CSI at the relay, in: Proceedings of the IEEE SPAWC, Helsinki, 2007, pp. 1–5.
[15] V. Havary-Nassab, S. Shahbazpanahi, A. Grami, Optimal distributed beamforming for two-way relay networks, IEEE Transactions on Signal Processing 58 (3) (2010) 1238–1250.
[16] Y. Shen, Z. Fei, C. Xing, J. Kuang, Beamforming design for multi-pair two-way AF MIMO relaying networks using quadratic programming, in: ICSPCC 2011, Xi'an, China, 2010.
[17] J. Choi, MMSE-based distributed beamforming in cooperative relay networks, IEEE Transactions on Communications 59 (5) (2011) 1346–1356.
[18] A.S. Behbahani, R. Merched, A.J. Eltawil, Optimizations of a MIMO relay network, IEEE Transactions on Signal Processing 56 (10) (2008) 5062–5073.
[19] H. Bolcskei, R.U. Nabar, O. Oyman, A.J. Paulraj, Capacity scaling laws in MIMO relay networks, IEEE Transactions on Wireless Communications 5 (6) (2006) 1433–1443.
[20] B. Wang, J. Zhang, A. Host-Madsen, On the capacity of MIMO relay channels, IEEE Transactions on Information Theory 51 (1) (2005) 29–43.
[21] C.-B. Chae, T.W. Tang, R.W. Health, S.-Y. Cho, MIMO relaying with linear processing for multiuser transmission in fixed relay networks, IEEE Transactions on Signal Processing 56 (2) (2008) 727–738.
[22] R. Zhang, C.C. Chai, Y.-C. Liang, Joint beamforming and power control for multiantenna relay broadcast channel with QoS constraints, IEEE Transactions on Signal Processing 57 (2) (2009) 726–737.
[23] S. Stefania, I. Toufik, M. Backer, LTE, The UMTS Long Term Evolution: From Theory to Practice, Wiley, 2009.
[24] A. Osseiran, et al., The road to IMT-advanced communication systems: state-of-the-art and innovation areas addressed by the WINNER + project [topics in radio communications], IEEE Communications Magazine 47 (6) (2009) 38–47.
[25] I.E. Telatar, Capacity of multi-antenna Gaussian channels, European Transactions on Telecommunications 10 (6) (1999) 585–595.
[26] S. Kay, Fundamental of Statistical Signal Processing: Estimation Theory, Prentice-Hall, Englewood Cliffs, NJ, 1993.
[27] E.G. Larsson, P. Stoica, Space–Time Block Coding for Wireless Communications, Cambridge University Press, 2003.
[28] D. Tse, P. Viswanath, Fundamentals of Wireless Communication, Cambridge University Press, 2005.
[29] H. Sampath, P. Stoica, A. Paulraj, Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion, IEEE Transactions on Communications 49 (12) (2006) 2198–2206.
[30] J. Zhang, Y. Wu, S. Zhou, J. Wang, Joint linear transmitter and receiver design for the downlink of multiuser MIMO, IEEE Communications Letters 9 (11) (2005) 991–993.

[31] S. Shi, M. Chubert, H. Boche, Rate optimization for multiuser MIMO systems with linear processing, IEEE Transactions on Signal Processing 56 (8) (2008) 4020–4030.
[32] S. Serbetli, A. Yener, Transceiver optimization for multiuser MIMO systems, IEEE Transactions on Signal Processing 52 (9) (2004) 214–226.
[33] M. Codreanu, A. Tolli, M. Juntti, M. Latva-aho, Joint design of Tx-Rx beamforming in MIMO downlink channel, IEEE Transactions on Signal Processing 55 (9) (2007) 4639–4655.
[34] S.S. Christensen, R. Agarwal, E. de Carvalho, J.M. Cioffi, Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design, IEEE Transactions on Wireless Communications 7 (12) (2008) 4792–4799.
[35] R. Hunger, M. Joham, W. Utschick, On the MSE-Duality of the broadcast channel and multiple access channel, IEEE Transactions on Signal Processing 57 (2) (2009) 698–713.
[36] S. Shi, M. Schubert, H. Boche, Downlink MMSE transceiver optimization for multiuser MIMO systems: duality and sum-MSE minimization, IEEE Transactions on Signal Processing 55 (11) (2007) 5436–5446.
[37] E. Matskani, N.D. Sidiropoulos, Z.-Q. Luo, L. Tassiulas, Convex approximation techniques for joint multiuser downlink beamforming and admission control, IEEE Transactions on Wireless Communications 7 (7) (2008) 2682–2693.
[38] A. Beck, Quadratic matrix programming, SIAM Journal on Optimization 17 (4) (2007) 1224–1238.
[39] S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004.
[40] C. Xing, S. Ma, Y.-C. Wu, Robust joint design of linear relay precoder and destination equalizer for dual-hop amplify-and-forward MIMO relay systems, IEEE Transactions on Signal Processing 58 (4) (2010) 2273–2283.
[41] L. Vandenberghe, S. Boyd, Semidefinite programming, SIAM Review 38 (1) (1996) 49–95.
[42] K.-K. Wong, A. Paulraj, R.D. Murch, Efficient high-performance decoding for overloaded MIMO antenna systems, IEEE Transactions on Wireless Communications 6 (5) (2007) 1833–1843.
[43] R. de Miguel, V. Gardasevic, R.R. Muller, F.F. Knudsen, On overloaded vector precoding for single-user MIMO channels, IEEE Transactions on Wireless Communications 9 (2) (2010) 745–753.
[44] C. Xing, Linear Mean-Square-Error Transceiver Design for Amplify-and-Forward Multiple Antenna Relaying Systems, Ph.D. Thesis, The University of Hong Kong, Hong Kong, July 2010.
[45] R.A. Horn, C.R. Johnson, Matrix Analysis, Cambridge University Press, 1985.
[46] G. Zheng, S. Ma, K.-K. Wong, T.-S. Ng, Robust beamforming in the MISO downlink with quadratic channel estimation and optimal training, IEEE Transactions on Wireless Communications 8 (3) (2009) 1067–1072.
[47] M. Schubert, H. Boche, Solution of the multi-user downlink beamforming problem with individual SINR constraints, IEEE Transactions on Vehicular Technology 53 (1) (2004) 18–28.
[48] M. Grant, S. Boyd, Y.Y. Ye, CVX: Matlab Software for Disciplined Convex Programming, Available at: 〈http://www.stanford.edu/boyd/cvx〉, V.1.0RC3, 2007.