Estimation and Compensation of CFO and I/Q Imbalance in OFDM Systems under Timing Ambiguity

Xun Cai, Yik-Chung Wu, Hai Lin, and Katsumi Yamashita

Abstract— **This paper proposes a joint carrier frequency offset (CFO) and I/Q imbalance estimation and compensation method for OFDM systems under timing uncertainty. A matrix representation of received signal with CFO, timing offset (TO) and I/Q imbalance is first derived. Then a least squares (LS) CFO estimator robust against TO and I/Q imbalance is proposed. The Cramér-Rao Bound for CFO is also derived. With the estimated CFO and equivalent channel, a compensation method for both frequency-dependent and frequency-independent I/Q imbalances is presented. Simulation results are provided to verify the performances of the proposed methods.**

Index Terms— **Carrier frequency offset (CFO), I/Q imbalance, channel estimation, timing offset, least squares estimation (LSE), Orthogonal frequency division multiplexing (OFDM).**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been applied to a wide range of communication systems because of its efficient utilization of available bandwidth and robustness against frequency-selective fading channels. However, it is known that OFDM system is very sensitive to carrier frequency offset (CFO), making CFO estimation and compensation crucial in OFDM systems. On the other hand, direct-conversion receiver (DCR) becomes more popular because of its merits in cost and power consumption. Nonetheless, it introduces the challenging I/Q imbalance problem, and results in mirrored inter-carrier interference in the received signal [1][2].

Estimation and compensation of CFO and frequency-independent (f-independent) I/Q imbalance in OFDM systems have been reported in [3]-[6]. However, for wide-band receiver used for OFDM signals, the more challenging frequency-dependent (f-dependent) I/Q imbalance also needs to be addressed. In [7], the f-dependent I/Q imbalance compensation under a small residual CFO has been discussed. On the other hand, [8] and [9] propose joint least squares (LS) estimation methods for CFO, f-independent and f-dependent I/Q imbalances using null-subcarriers and a specially designed training sequence, respectively.

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Notice that all of the above methods assume perfect timing synchronization, which is very difficult to achieve in practice due to the plateau in the metrics of the timing synchronization algorithms under frequency selective fading channel [10]-[12]. Recently, [13] analyzes the effect of timing offset (TO) to estimation and compensation of I/Q imbalance, but only f-independent imbalance is considered.

In this paper, the problem of joint CFO and I/Q imbalance (both f-dependent and f-independent) estimation and compensation under timing ambiguity is addressed. In particular, with all the effects of CFO, multipath channel, I/Q imbalance and timing offset intertwined together, the system model in matrix form is first established via non-trivial derivation. Then, the LS estimation of CFO and the effective channel is presented, and the Cramér-Rao Bound (CRB) of CFO is also derived. Furthermore, a novel CFO, channel, I/Q imbalance and TO compensation method is proposed. Relationship between the proposed compensation method and that based on null-subcarriers [8] is revealed. The proposed estimation and compensation algorithms do not impose any constraint on the training structure, and therefore are very flexible. Simulation results are presented to show the effectiveness of the proposed estimation and compensation methods.

Notation: Upper and lower boldface letters represent matrices and vectors respectively. $\mathbf{I}_N$ indicates the $N \times N$ identity matrix. $\Re(\cdot)$ and $\Im(\cdot)$ take the real and imaginary parts of a complex quantity. The operation $\mathrm{tr}(\mathbf{A})$ takes the trace of matrix $\mathbf{A}$. Operation $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix with the elements of $\mathbf{x}$ located along the main diagonal. Superscript $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^*$ denote Hermitian, transpose and conjugation, respectively. Notation $\otimes$ denotes convolution operation. MATLAB notation is employed throughout the paper to specify the rows and columns of a matrix.

II. SYSTEM MODEL

A. I/Q Imbalance Model

In this paper, we employ the DCR model shown in Figure 1a, where the f-independent I/Q imbalance is characterized by an amplitude mismatch α and a phase mismatch ϕ, while the f-dependent imbalance is modeled by two real lowpass filters in I and Q branch with frequency response $G_I(f)$, $G_Q(f)$ and impulse response $\mathbf{g}_I$, $\mathbf{g}_Q$ respectively. $G_I(f)$ and $G_Q(f)$ are assumed to be zero for $|f| > B/2$, where B is the bandwidth of the system.

Denoting $s(t)$ as the baseband transmitted signal, $h(t)$ as the baseband equivalent channel, the RF signal $r_{\text{RF}}(t)$ is given by [8]

$$r_{\text{RF}}(t) = \Re\{[s(t) \otimes h(t)] \cdot e^{j2\pi f_c t}\}, \tag{1}$$

where f_c is the carrier frequency. According to [8] and derivations in [9], [14], at the receiver, the down-converted signal $r(t)$ is

$$\begin{aligned} r(t) = &\{e^{j2\pi\Delta f t} \cdot [s(t) \otimes h(t)]\} \otimes c_1(t) \\ &+ \{e^{-j2\pi\Delta f t} \cdot [s^*(t) \otimes h^*(t)]\} \otimes c_2(t) + w(t), \end{aligned} \tag{2}$$

where Δf is the frequency offset, and

$$c_1(t) = \frac{1}{2}\mathcal{F}^{-1}\{G_I(f) + \alpha e^{-j\phi} G_Q(f)\}, \tag{3}$$

$$c_2(t) = \frac{1}{2}\mathcal{F}^{-1}\{G_I(f) - \alpha e^{j\phi} G_Q(f)\}, \tag{4}$$

$$w(t) = z(t) \otimes c_1(t) + z^*(t) \otimes c_2(t), \tag{5}$$

with $\mathcal{F}^{-1}\{\cdot\}$ being the inverse Fourier transform, and $z(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2.

Suppose $c_1(t)$, $c_2(t)$ and $h(t)$ span the periods of L_1T_s, L_2T_s and L_hT_s, where T_s is the sampling period satisfying Nyquist sampling theorem, we obtain the discrete-time version of signal in (2) as

$$r(n) = \overleftarrow{r}(n) + \overrightarrow{r}(n) + w(n), \tag{6}$$

where $\overleftarrow{r}(n) = \{e^{j2\pi\Delta f n T_s}[s(n) \otimes h(n)]\} \otimes c_1(n)$ and $\overrightarrow{r}(n) = \{e^{-j2\pi\Delta f n T_s}[s^*(n) \otimes h^*(n)]\} \otimes c_2(n)$ with $c_1(n)$, $c_2(n)$, $h(n)$ and $w(n)$ being the sampled version of $c_1(t)$, $c_2(t)$, $h(t)$ and $w(t)$ respectively. Notice that $\overleftarrow{r}(n)$ is the desired signal component and $\overrightarrow{r}(n)$ is the image interference caused by I/Q imbalance [8].

B. Received OFDM Signal under TO and Unknown Channel Length

In this paper, we consider a packet-based OFDM system, in which each packet consists of a training OFDM symbol and then followed by a number of OFDM data symbols. For an OFDM system with N subcarriers, denote $s(n)$ as the sample from $\mathbf{F}^H\mathbf{S}$ under perfect timing synchronization, where $\mathbf{F}$ is $N \times N$ DFT matrix with $\mathbf{F}(k,l) = \frac{1}{\sqrt{N}}e^{-j2\pi kl/N}$ and $\mathbf{S} = \text{diag}[S_0, S_1, \cdots, S_{N-1}]$ with S_m being the data (or pilot) carried by the m^{th} subcarrier. Because of the cyclic prefix (CP) inserted ahead of each OFDM symbol, the linear convolution in (6) becomes circular convolution. Then $s(n) \otimes h(n)$ can be obtained from the sample of $\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h}$, where $\mathbf{h} \triangleq [h(0), h(1), \cdots, h(L_h-1)]^T$ and $\mathbf{F}_{L_h}$ denotes the first L_h columns of $\mathbf{F}$. At the receiver, a correlator-based timing synchronization algorithm is used (e.g., [10],[11]) such that the inter-symbol interference (ISI) free region can be identified. Assuming the FFT window starts at θ_0 position from the ideal one (but still within the ISI free region)[1], collecting N samples of $\overleftarrow{r}(n)$ gives

$$\overleftarrow{\mathbf{r}} = \mathbb{C}_1\mathbf{\Gamma}_N(\varepsilon)\mathbf{T}_{\theta_0}\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h}, \tag{7}$$

[1]Only integer delay is explicitly modeled here. Fractional delay is treated as part of the multipath channel.

where $\overleftarrow{\mathbf{r}} = [\overleftarrow{r}(0), \overleftarrow{r}(1), ..., \overleftarrow{r}(N-1)]^T$, $\mathbf{\Gamma}_k(\varepsilon) = \text{diag}[1, e^{j2\pi\varepsilon/N}, \ldots, e^{j2\pi\varepsilon(k-1)/N}]$ with $\varepsilon = \Delta f \cdot N/B$, $\mathbf{T}_{\theta_0} = [\mathbf{e}_{\theta_0+1}, \cdots, \mathbf{e}_N, \mathbf{e}_1, \cdots, \mathbf{e}_{\theta_0}]$ with $\mathbf{e}_i = \mathbf{I}_N(:, i)$ and $\mathbb{C}_1$ is an $N \times N$ circulant matrix with the first column given by $[c_1(0), c_1(1), \cdots, c_1(L_1-1), \underbrace{0, \cdots, 0}_{(N-L_1)\text{zeros}}]^T$. That is,

$$\mathbb{C}_1 = \begin{bmatrix} c_1(0) & 0 & \cdots & c_1(1) \\ \vdots & c_1(0) & \ddots & \vdots \\ c_1(L_1-2) & \vdots & \ddots & c_1(L_1-1) \\ c_1(L_1-1) & c_1(L_1-2) & \ddots & 0 \\ 0 & c_1(L_1-1) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & c_1(0) \end{bmatrix}_{N\times N}. \tag{8}$$

Reversing the positions of $\mathbb{C}_1$ and $\mathbf{\Gamma}_N(\varepsilon)$ in (7), it can be easily shown that

$$\overleftarrow{\mathbf{r}} = \mathbf{\Gamma}_N(\varepsilon)\tilde{\mathbb{C}}_1\mathbf{T}_{\theta_0}\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h}, \tag{9}$$

where $\tilde{\mathbb{C}}_1$ is obtained by replacing $c_1(l)$ in $\mathbb{C}_1$ by $c_1(l)e^{-j2\pi\varepsilon l/N}$.

Since $\tilde{\mathbb{C}}_1$, $\mathbf{T}_{\theta_0}$ and $\mathbf{F}^H\mathbf{S}\mathbf{F}$ are circulant matrices, their positions can be exchanged [15], and we can rewrite (9) as

$$\overleftarrow{\mathbf{r}} = \mathbf{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}\mathbf{T}_{\theta_0}\tilde{\mathbb{C}}_1\mathbf{F}^H\mathbf{F}_{L_h}\mathbf{h}. \tag{10}$$

Note that $\mathbf{F}^H\mathbf{F}_{L_h} = [\mathbf{I}_{L_h} \quad \mathbf{0}_{L_h\times(N-L_h)}]^T$, where $\mathbf{0}_{N\times L}$ is a $N \times L$ matrix consists of zero elements. Since the last $N - L_h$ rows of $\mathbf{F}^H\mathbf{F}_{L_h}$ are zeros, only the first L_h columns of $\tilde{\mathbb{C}}_1$ should be retained in (10) and we have

$$\overleftarrow{\mathbf{r}} = \mathbf{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}\mathbf{T}_{\theta_0}\tilde{\mathbb{C}}_1(:, 1:L_h)\mathbf{h}. \tag{11}$$

Looking into the structure of $\tilde{\mathbb{C}}_1(:, 1:L_h)$, it is found that only the first $L_a \triangleq L_1 + L_h - 1$ rows have non-zeros elements. Therefore, (11) can be further simplified as

$$\overleftarrow{\mathbf{r}} = \mathbf{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}\mathbf{T}_{\theta_0}(:, 1:L_a)\tilde{\mathbb{C}}_1(1:L_a, 1:L_h)\mathbf{h}. \tag{12}$$

Assuming $\theta_0 + L_a$ is smaller than the length of CP (L_{cp}), we only need to retain the first L_{cp} rows of $\mathbf{T}_{\theta_0}(:, 1:L_a)$. Therefore,

$$\overleftarrow{\mathbf{r}} = \mathbf{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_{\text{cp}}}\underbrace{\mathbf{T}_{\theta_0}(1:L_{\text{cp}}, 1:L_a)\tilde{\mathbb{C}}_1(1:L_a, 1:L_h)\mathbf{h}}_{\triangleq\mathbf{a}}, \tag{13}$$

with $\mathbf{a}$ being defined as the equivalent channel consisting of the effect of TO, channel and I/Q imbalance parameters.

With similar derivations and the assumption that $\theta_0 + L_b \leqslant L_{\text{cp}}$ with $L_b = L_2 + L_h - 1$, the expression for $\overrightarrow{\mathbf{r}}$ is

$$\overrightarrow{\mathbf{r}} = \mathbf{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}^*_{L_{\text{cp}}}\mathbf{b}, \tag{14}$$

where $\mathbf{b} \triangleq \mathbf{T}_{\theta_0}(1 : L_{\text{cp}}, 1 : L_b)\tilde{\mathbb{C}}_2(1 : L_b, 1 : L_h)\mathbf{h}^*$, and $\tilde{\mathbb{C}}_2$ is an $N \times N$ circulant matrix with the first column given by $[c_2(0), c_2(1)e^{j2\pi\varepsilon/N}, \cdots, c_2(L_2-1)e^{j2\pi\varepsilon(L_2-1)/N}, 0, \cdots, 0]^T$ and having the same structure as

$\mathbb{C}_1$ in (8). From (13) and (14), we can obtain the model for the received signal as

$$\mathbf{r} = \mathbf{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_{\text{cp}}}\mathbf{a} + \mathbf{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}^*_{L_{\text{cp}}}\mathbf{b} + \mathbf{w}, \quad (15)$$

$$= \underbrace{\left[\begin{array}{cc} \mathbf{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_{\text{cp}}} & \mathbf{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}^*_{L_{\text{cp}}} \end{array}\right]}_{\triangleq \mathbf{A}(\varepsilon)} \underbrace{\left[\begin{array}{c} \mathbf{a} \\ \mathbf{b} \end{array}\right]}_{\triangleq \mathbf{d}} + \mathbf{w}, \quad (16)$$

where $\mathbf{w} \triangleq [w(0), w(1), \cdots, w(N-1)]^T$.

Remark: Note that $\mathbf{a}$ and $\mathbf{b}$ can also be expressed as

$$\mathbf{a} = [\mathbf{0}^T_{\theta_0\times 1} \quad [\mathbf{h} \otimes \mathbf{\Gamma}_{L_1}(-\varepsilon)\mathbf{c}_1]^T \quad \mathbf{0}^T_{(L_{\text{cp}}-\theta_0-L_a)\times 1}]^T, \quad (17)$$

$$\mathbf{b} = [\mathbf{0}^T_{\theta_0\times 1} \quad [\mathbf{h}^* \otimes \mathbf{\Gamma}_{L_2}(\varepsilon)\mathbf{c}_2]^T \quad \mathbf{0}^T_{(L_{\text{cp}}-\theta_0-L_b)\times 1}]^T, \quad (18)$$

where $\mathbf{c}_1 = [c_1(0), c_1(1), \cdots, c_1(L_1-1)]^T$ and $\mathbf{c}_2 = [c_2(0), c_2(1), \cdots, c_2(L_2-1)]^T$. In other words, the composite channels for $\overleftarrow{\mathbf{r}}$ and $\overrightarrow{\mathbf{r}}$ are the delayed version of the actual channel convolved with the CFO rotated $\mathbf{c}_1$ and $\mathbf{c}_2$, respectively.

III. CFO Estimation under TO and I/Q Imbalance

From (5), it is noticed that $\mathbf{w}$ depends on the unknown I/Q imbalance parameters $\mathbf{c}_1$ and $\mathbf{c}_2$. Therefore, maximum likelihood (ML) estimator is very difficult to derive. Here, we treat the whole equivalent channel $\mathbf{a}$ and $\mathbf{b}$ in (16) as unknowns and employ the LS method to estimate the CFO.

From (16), the LS estimates of ε and $\mathbf{d}$ are obtained by minimizing the objective function

$$J_{\text{CFO}}(\varepsilon, \mathbf{d}) = \|\mathbf{r} - \mathbf{A}(\varepsilon)\mathbf{d}\|^2. \quad (19)$$

Because $\mathbf{d}$ is a linear parameter, the LS solution for $\mathbf{d}$ in terms of ε can be obtained by first differentiating J_{CFO} with respect to (w.r.t.) $\mathbf{d}$, setting it to zero and then solving the obtained equation. With the above steps, we obtain

$$\hat{\mathbf{d}} = [\mathbf{A}^H(\varepsilon)\mathbf{A}(\varepsilon)]^{-1}\mathbf{A}^H(\varepsilon)\mathbf{r}. \quad (20)$$

Substituting $\hat{\mathbf{d}}$ into (19), we obtain the LS estimator of ε as [16]

$$\hat{\varepsilon} = \arg\max_{\tilde{\varepsilon}}\{\mathbf{r}^H\mathbf{A}(\tilde{\varepsilon})[\mathbf{A}^H(\tilde{\varepsilon})\mathbf{A}(\tilde{\varepsilon})]^{-1}\mathbf{A}^H(\tilde{\varepsilon})\mathbf{r}\}, \quad (21)$$

where $\tilde{\varepsilon}$ is the trial value of ε. $\hat{\varepsilon}$ can be found by grid search or gradient search. After $\hat{\varepsilon}$ is obtained, $\hat{\mathbf{d}}$ is obtained by plugging $\hat{\varepsilon}$ into (20). Notice that the estimation method proposed in this section is performed only at OFDM blocks carrying training sequences. The CRB for ε has also been derived and is detailed in the Appendix.

In order for this estimator to work, matrix $\mathbf{A}(\varepsilon)$ should be a tall matrix, which means the number of rows should be no less than the number of columns. This requirement is equivalent to $2L_{\text{cp}} \leqslant N$. In practice, N is usually much larger than $2L_{\text{cp}}$, which makes this requirement valid in most cases.

For the CFO estimation in (21), the complexity is dominated by $[\mathbf{A}^H(\tilde{\varepsilon})\mathbf{A}(\tilde{\varepsilon})]^{-1}$, which costs complexity $O(4L_{cp}^2N)$ for computation of $\mathbf{A}^H(\tilde{\varepsilon})\mathbf{A}(\tilde{\varepsilon})$ and $O(8L_{cp}^3)$ for the inverse operation. Since in practical system, the number of subcarriers N is much larger than the length of CP L_{cp}, and assuming N_ω is the number of grid-points to be searched for the CFO, the total complexity order is $O(NL_{cp}^2N_\omega)$.

IV. I/Q Imbalance Compensation

A. Direct Demodulation

After $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and $\hat{\varepsilon}$ are obtained from the proposed estimator, the system model in (16) can be rewritten as

$$\underbrace{\left[\begin{array}{c} \Re\{\mathbf{r}\} \\ \Im\{\mathbf{r}\} \end{array}\right]}_{\triangleq \mathbf{r}_c} = \underbrace{\left[\begin{array}{cc} \Re\{\mathbf{D}_1+\mathbf{D}_2\} & -\Im\{\mathbf{D}_1-\mathbf{D}_2\} \\ \Im\{\mathbf{D}_1+\mathbf{D}_2\} & \Re\{\mathbf{D}_1-\mathbf{D}_2\} \end{array}\right]}_{\triangleq \mathbf{D}_c} \underbrace{\left[\begin{array}{c} \Re\{\mathbf{s}_f\} \\ \Im\{\mathbf{s}_f\} \end{array}\right]}_{\triangleq \mathbf{s}_c} + \underbrace{\left[\begin{array}{c} \Re\{\mathbf{w}\} \\ \Im\{\mathbf{w}\} \end{array}\right]}_{\triangleq \mathbf{w}_c}, \quad (22)$$

where $\mathbf{s}_f = [S_0, S_1, ..., S_{N-1}]^T$, $\mathbf{D}_1 = \mathbf{\Gamma}_N(\hat{\varepsilon})\mathbf{F}^H\text{diag}(\mathbf{F}_{L_a}\hat{\mathbf{a}})$ and $\mathbf{D}_2 = \mathbf{\Gamma}_N(-\hat{\varepsilon})\mathbf{F}\text{diag}(\mathbf{F}^*_{L_b}\hat{\mathbf{b}})$. From this equation, it is clear that the real and imaginary part of the transmitted data can be estimated using

$$\hat{\mathbf{s}}_c = \text{demod}\{[\mathbf{D}_c^H\mathbf{D}_c]^{-1}\mathbf{D}_c\mathbf{r}_c\}, \quad (23)$$

where demod$\{\cdot\}$ denotes the demodulation operation according to the symbol constellation.

This demodulation method can recover the transmitted data under TO, CFO and I/Q imbalance. However, from (23), it is noticed that this demodulation is done blockwise with an inverse operation of a large dimensional matrix $\mathbf{D}_c$. As a result, its computational complexity is high compared to the traditional tone-by-tone demodulation in OFDM system. In the following, we propose an I/Q imbalance compensation method that compensate the I/Q imbalance before demodulation, and therefore tone-by-tone demodulation is feasible.

B. Proposed Compensation Method

The main idea of the proposed compensation method is to minimize the difference between the compensated signal and a target signal, which is chosen to be closely related to the received signal but without I/Q imbalance. To fulfill the requirements, we propose to use target signal

$$\mathbf{r}_t = \mathbf{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_{\text{cp}}}\mathbf{h}_{\text{eq}}, \quad (24)$$

where $\mathbf{h}_{\text{eq}} = \mathbf{a} + \mathbf{b}^*$ is the equivalent channel.

The physical meaning of the equivalent channel can be explained as follows. Without loss of generality, vectors $\mathbf{g}_I$, $\mathbf{g}_Q$, $\mathbf{c}_1$ and $\mathbf{c}_2$ are assumed to be of the same length and is denoted by L_c. From (3) and (4), we have $\mathbf{c}_1 = \frac{1}{2}(\mathbf{g}_I + \alpha e^{-j\phi}\cdot\mathbf{g}_Q)$ and $\mathbf{c}_2 = \frac{1}{2}(\mathbf{g}_I - \alpha e^{j\phi}\cdot\mathbf{g}_Q)$. Then from (17) and (18), $\mathbf{h}_{\text{eq}} = \mathbf{a} + \mathbf{b}^*$ can be derived as

$$\mathbf{h}_{\text{eq}} = [\mathbf{0}^T_{\theta_0\times 1} \quad \mathbf{h}^T \otimes [\mathbf{\Gamma}_{L_c}(-\varepsilon)\mathbf{g}_I]^T \quad \mathbf{0}^T_{(L_{\text{cp}}-\theta_0-L_h-L_c+1)\times 1}]^T. \quad (25)$$

This shows that the equivalent channel is the time delayed version of actual channel convolved with the CFO rotated $\mathbf{g}_I$. Therefore, the contribution of the Q branch filter $\mathbf{g}_Q$ is eliminated. In practice, $\mathbf{h}_{\text{eq}}$ is not known a prior. But fortunately, $\mathbf{a}$ and $\mathbf{b}$ have been estimated in (20) and can be used to generate $\mathbf{h}_{\text{eq}}$.

According to [9], down-converted signal with CFO and I/Q imbalance can be effectively compensated using the structure in Figure 1b, where $\mathbf{x} = [x(0), x(1), \cdots, x(L_x-1)]^T$ is a filter and β is an amplification coefficient. With this structure, the expression for I/Q imbalance compensated signal can be written as

$$\ddot{\mathbf{r}} = \mathbf{r}_I + j(\mathbf{R}_Q\mathbf{x} + \mathbf{r}_I\beta), \quad (26)$$

where $\mathbf{r}_I = \Re\{\mathbf{r}\}$ and $\mathbf{R}_Q$ is an $N \times L_x$ circulant matrix with the first column given by $[\Im\{r(0)\}, \Im\{r(1)\}, \cdots, \Im\{r(N-1)\}]^T$.

We want to design $\mathbf{x}$ and β such that the output $\ddot{\mathbf{r}}$ matches the target signal as close as possible. That is,

$$(\hat{\mathbf{x}}, \hat{\beta}) = \arg\min_{\mathbf{x},\beta} J_{\text{IQ}}(\mathbf{x}, \beta) \triangleq \|\hat{\mathbf{r}}_t - \ddot{\mathbf{r}}\|^2, \quad (27)$$

where $\hat{\mathbf{r}}_t = \mathbf{r}_t|_{\mathbf{a}=\hat{\mathbf{a}},\mathbf{b}=\hat{\mathbf{b}},\varepsilon=\hat{\varepsilon}}$. Differentiating the objective function in (27) w.r.t. $\mathbf{x}$ and β, we get

$$\frac{\partial J_{\text{IQ}}}{\partial \beta} = -\mathbf{r}_I^H[\mathbf{t} - (\mathbf{R}_Q\mathbf{x} + \mathbf{r}_I\beta)], \quad (28)$$

$$\frac{\partial J_{\text{IQ}}}{\partial \mathbf{x}} = -\mathbf{R}_Q^H[\mathbf{t} - (\mathbf{R}_Q\mathbf{x} + \mathbf{r}_I\beta)], \quad (29)$$

where $\mathbf{t} = \Re\{\hat{\mathbf{r}}_t\}$. Setting the above two equations to zero and solving the obtained equations, we get the optimal solutions

$$\hat{\beta} = \frac{\mathbf{r}_I^H\mathbf{R}_Q(\mathbf{R}_Q^H\mathbf{R}_Q)^{-1}\mathbf{R}_Q^H\mathbf{t} - \mathbf{r}_I^H\mathbf{t}}{\mathbf{r}_I^H\mathbf{R}_Q(\mathbf{R}_Q^H\mathbf{R}_Q)^{-1}\mathbf{R}_Q^H\mathbf{r}_I - \mathbf{r}_I^H\mathbf{r}_I}, \quad (30)$$

$$\hat{\mathbf{x}} = (\mathbf{R}_Q^H\mathbf{R}_Q)^{-1}\left[\mathbf{R}_Q^H\mathbf{t} - \mathbf{R}_Q^H\mathbf{r}_I\hat{\beta}\right]. \quad (31)$$

In the I/Q imbalance compensation step, the computation is dominated by $(\mathbf{R}_Q^H\mathbf{R}_Q)^{-1}$ which costs $O(L_x^2 N)$ for computation of $\mathbf{R}_Q^H\mathbf{R}_Q$ and $O(L_x^3)$ for the inverse operation. Since N is much larger than L_x, the complexity of I/Q compensation is $O(L_x^2 N)$. Comparing the complexity of CFO estimation and I/Q imbalance compensation, it is obvious that the CFO estimation costs more computation, and I/Q imbalance compensation is a relative efficient step.

C. Relationship with Null Subcarrier Based Compensation Method [8]

The objective function in (27) is expressed in time domain. In frequency domain, it can be written as

$$J_{\text{IQ}}(\mathbf{x}, \beta) = \|\mathbf{F}\mathbf{\Gamma}_N(-\hat{\varepsilon})\hat{\mathbf{r}}_t - \mathbf{F}\mathbf{\Gamma}_N(-\hat{\varepsilon})\ddot{\mathbf{r}}\|^2. \quad (32)$$

Note that this modification does not change the objective function since $\mathbf{F}$ is a unitary matrix (i.e., $\mathbf{F}^H\mathbf{F} = \mathbf{I}$). If the training sequence in frequency domain is chosen to have both unloaded and loaded subcarriers, and let $\mathbf{V}$ and $\mathbf{W}$ denote the columns of $\mathbf{F}^H$ corresponding to unloaded and loaded subcarriers respectively, (32) becomes

$$J_{\text{IQ}}(\mathbf{x}, \beta) = J_1 + J_2, \quad (33)$$

where $J_1 = \|\mathbf{V}^H\mathbf{\Gamma}_N(-\hat{\varepsilon})\hat{\mathbf{r}}_t - \mathbf{V}^H\mathbf{\Gamma}_N(-\hat{\varepsilon})\ddot{\mathbf{r}}\|^2$ and $J_2 = \|\mathbf{W}^H\mathbf{\Gamma}_N(-\hat{\varepsilon})\hat{\mathbf{r}}_t - \mathbf{W}^H\mathbf{\Gamma}_N(-\hat{\varepsilon})\ddot{\mathbf{r}}\|^2$, since $\mathbf{F}^H\mathbf{F} = \mathbf{V}\mathbf{V}^H + \mathbf{W}\mathbf{W}^H = \mathbf{I}_N$.

Since the target values for the unloaded subcarriers are zeros (i.e., $\mathbf{V}^H\mathbf{\Gamma}_N(-\hat{\varepsilon})\hat{\mathbf{r}}_t = \mathbf{0}_{N\times 1}$), J_1 reduces to $J_1 = \|\mathbf{V}^H\mathbf{\Gamma}_N(-\hat{\varepsilon})\ddot{\mathbf{r}}\|^2$, which is the same as the I/Q imbalance compensation objective function in [8]. Therefore, [8] is a special case of the proposed method when only the unloaded subcarriers are considered.

V. Simulation Results and Discussions

Simulation results are presented to illustrate the performances of proposed estimation and compensation methods. The parameters of the simulated OFDM system are selected as $B = 20$MHz, $N = 128$, $L_{\text{cp}} = 8$, and 64-QAM is employed. The channel is chosen to have three paths (i.e., $L_h = 3$) with an exponential decaying power profile. Each path is generated as independent complex Gaussian random variable (R.V.). The severe I/Q imbalance setting in [9], where the amplitude mismatch $\alpha = 1$dB, phase mismatch $\phi = 5°$ and f-dependent I/Q imbalance $\mathbf{g}_I = [0, 1, 0.1]^T$, $\mathbf{g}_Q = [0.1, 1, 0]^T$, is assumed. The length of the compensation filter is $L_x = 5$. In each simulation run, the CFO parameter ε is uniformly drawn from $[-0.5, 0.5]$ and the timing offset is uniformly drawn from the ISI-free region. Noise is generated in time domain as independent complex Gaussian R.V. Each point in the figures is obtained by averaging 5000 trials. The signal-to-noise ratio (SNR) in the figures is defined as the ratio of symbol energy per subcarrier over noise power.

We consider a full training case, and the training on each subcarier is generated as independent complex Gaussian R.V. with zero mean and unit variance. The estimation and compensation methods in [8] and [9] are also implemented for comparison. In order to have fair comparison, the training sequences in [8] and [9] are scaled such that they have the same energy as that in the proposed method.

Figure 2 compares the mean square error (MSE) of $\hat{\varepsilon}$ versus different SNRs for different methods. The CRB is also shown in the figure. From this figure, it is clear that the proposed method performs better than the other two methods and basically achieves the CRB, even though the TO θ_0 and channel length are unknown to the estimator. The performance improvement over method in [8] is because of full-utilization of training sequence; while the improvement over the method in [9] is due to LS estimation method.

Figure 3 shows the I/Q imbalance compensation results in terms of bit-error-rate (BER). In this figure, we can observe that the compensation method in [9] is ineffective due to the TO. On the other hand, the proposed compensation method performs better than that in [8]. This is because the proposed method allows pilots to be placed in all subcarriers, results in better estimation of CFO and equivalent channel, and subsequently gives better compensation results. Moreover, the performance of the proposed method basically is the same as that of direct demodulation, and they are close to the ideal case, which refers to the system with no CFO, no I/Q imbalance and with perfectly known channel. This reveals that the proposed method can effectively compensate I/Q imbalance while enabling the traditional and low complexity tone-by-tone demodulation.

In addition to the I/Q imbalance filters specified in the first paragraph of this section, we have also run simulations on BER with $\mathbf{g}_I = [0, 1, 0.2, 0.1, 0.01]^T$ and $\mathbf{g}_Q = [0.1, 1, 0, 0, 0]^T$, which is even more challenging. The length of the compensation filter is $L_x = 15$. It is found that BER performance of the proposed method in this case is similar to that in Figure 3 and the same conclusions can be drawn.

In order to understand the reason that causes the performance gap between the proposed compensation method and the ideal case in the BER, Figure 4 presents the BER curves with different amount of CFO and channel state information. In particular, as can be seen from the figure, if the CFO is perfectly known (but with channel and I/Q imbalance unknown), the BER of the resultant system is still the same as the proposed method estimating and compensating CFO, channel and I/Q imbalance. This shows that the bottleneck is not at the CFO estimation and compensation. On the other hand, if the CFO, I/Q imbalance and channel are perfectly known (but still the proposed compensation structure is used to recover the data), it can be seen that the BER performance is very close to that of ideal case. This means that the proposed I/Q imbalance structure is not the main cause of BER degradation. The main cause of degradation is from the imperfect knowledge of IQ imbalance and channel. Therefore, if we want to have the BER curve of the proposed method further approach the ideal case, we can increase the training sequence length (e.g., using two or more OFDM symbols as training) so as to increase the accuracy of channel and I/Q imbalance estimation.

VI. CONCLUSIONS

In this paper, CFO and I/Q imbalance estimation and compensation problems under timing uncertainty in OFDM systems were addressed. The model of received signal with CFO, TO and I/Q imbalance in OFDM system was first derived. A CFO estimation algorithm robust to I/Q imbalance and TO was proposed. The CRB for the CFO was also derived, and simulation results showed that the proposed CFO estimator performs very close to CRB. Furthermore, a novel I/Q imbalance compensation method was presented. With the estimated CFO and equivalent channel, closed-form solutions for the compensation parameters were obtained. Simulation results verified the superior performance of the proposed compensation method.

APPENDIX

To assess the accuracy of the CFO estimator, we will derive the CRB as a reference in this Appendix. Here, we derive the CRB under the assumption that there is no TO and the channel length is known. This assumption means more information is used in CRB derivation than in CFO estimator proposed in Section III, and hence the resultant CRB is still a valid lower bound for the considered system.

First, the unknown parameters are defined to be $\boldsymbol{\eta} \triangleq [\varepsilon, \Re\{\mathbf{c}_1\}^T, \Im\{\mathbf{c}_1\}^T, \Re\{\mathbf{c}_2\}^T, \Im\{\mathbf{c}_2\}^T, \Re\{\mathbf{h}\}^T, \Im\{\mathbf{h}\}^T]^T$. Denote $\boldsymbol{\mu} = \mathbf{r} - \mathbf{w}$, the $(m,n)^{\text{th}}$ entry of the Fisher Information matrix $\mathbf{J}$ can be computed as [16]

$$[\mathbf{J}]_{mn} = 2\Re\left\{\frac{\partial \boldsymbol{\mu}^H}{\partial \eta_i}\mathbf{R}_{\mathbf{w}}^{-1}\frac{\partial \boldsymbol{\mu}}{\partial \eta_j^T}\right\} + \mathrm{tr}\left(\mathbf{R}_{\mathbf{w}}^{-1}\frac{\partial \mathbf{R}_{\mathbf{w}}}{\partial \eta_i}\mathbf{R}_{\mathbf{w}}^{-1}\frac{\partial \mathbf{R}_{\mathbf{w}}}{\partial \eta_j}\right), \tag{34}$$

where $[\mathbf{J}]_{mn}$ denotes the $(m,n)^{\text{th}}$ element of $\mathbf{J}$, η_i denotes the i^{th} element of $\boldsymbol{\eta}$, and

$$\mathbf{R}_{\mathbf{w}} = \mathbb{E}[\mathbf{w}\mathbf{w}^H] = \mathbb{C}_1\mathbf{R}_{\mathbf{z}}\mathbb{C}_1^H + \mathbb{C}_2\mathbf{R}_{\mathbf{z}}\mathbb{C}_2^H, \tag{35}$$

with $\mathbf{R}_{\mathbf{z}} = \sigma^2\mathbf{I}_N$ and $\mathbb{C}_2$ being an $N \times N$ circulant matrix with the first column given by $[c_2(0), c_2(1), \cdots, c_2(L_2-1), 0, \cdots, 0]^T$ and having the same structure as $\mathbb{C}_1$ in (8).

To obtain the individual component of (34), we first need to obtain an expression for $\boldsymbol{\mu}$. From (7), setting $\theta_0 = 0$, we have $\overleftarrow{\mathbf{r}} = \mathbb{C}_1\boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h}$. With the corresponding expression for $\overrightarrow{\mathbf{r}}$, we obtain

$$\boldsymbol{\mu} = \mathbb{C}_1\boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h} + \mathbb{C}_2\boldsymbol{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}_{L_h}^*\mathbf{h}^*. \tag{36}$$

Differentiating (36) w.r.t. ε, $\Re\{\mathbf{h}\}$ and $\Im\{\mathbf{h}\}$ gives

$$\begin{aligned}\frac{\partial \boldsymbol{\mu}}{\partial \varepsilon} &= j \cdot \frac{2\pi}{N} \cdot [\mathbb{C}_1\mathbf{D}\boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h} \\ &\quad - \mathbb{C}_2\mathbf{D}\boldsymbol{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}_{L_h}^*\mathbf{h}^*], \quad (37)\\ \frac{\partial \boldsymbol{\mu}}{\partial \Re\{\mathbf{h}\}^T} &= \mathbb{C}_1\boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h} + \mathbb{C}_2\boldsymbol{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}_{L_h}^*\mathbf{h}^*, \quad (38)\\ \frac{\partial \boldsymbol{\mu}}{\partial \Im\{\mathbf{h}\}^T} &= j \cdot [\mathbb{C}_1\boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_h}\mathbf{h} - \mathbb{C}_2\boldsymbol{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}_{L_h}^*\mathbf{h}^*]. \quad (39)\end{aligned}$$

On the other hand, setting $\theta_0 = 0$ in (11), we have $\overleftarrow{\mathbf{r}} = \boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}\tilde{\mathbb{C}}_1(1:N, 1:L_h)\mathbf{h}$. Changing the positions of $\mathbf{h}$ and $\tilde{\mathbb{C}}_1(1:N, 1:L_h)$, we get

$$\overleftarrow{\mathbf{r}} = \boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}\mathbb{H}(1:N, 1:L_1)\boldsymbol{\Gamma}_{L_1}(-\varepsilon)\mathbf{c}_1, \tag{40}$$

where $\mathbb{H}$ is an $N \times N$ circulant matrix with the first column given by $[h(0), h(1), \cdots, h(L_h-1), 0, \cdots, 0]^T$ and having the same structure as $\mathbb{C}_1$ in (8). Since only the first L_a rows of $\mathbb{H}(1:N, 1:L_h)$ have non-zero elements, disregarding the last $N - L_a$ rows gives

$$\overleftarrow{\mathbf{r}} = \boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_a}\mathbb{H}(1:L_a, 1:L_h)\boldsymbol{\Gamma}_{L_1}(-\varepsilon)\mathbf{c}_1. \tag{41}$$

Applying similar operations to $\overrightarrow{\mathbf{r}}$, it can be proved that an alternative expression for $\boldsymbol{\mu}$ is given by

$$\begin{aligned}\boldsymbol{\mu} &= \boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_a}\mathbb{H}(1:L_a, 1:L_1)\boldsymbol{\Gamma}_{L_1}(-\varepsilon)\mathbf{c}_1 \\ &\quad + \boldsymbol{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}_{L_b}^*\mathbb{H}^*(1:L_b, 1:L_2)\boldsymbol{\Gamma}_{L_2}(\varepsilon)\mathbf{c}_2. \quad (42)\end{aligned}$$

Differentiating (42) w.r.t. $\Re\{\mathbf{c}_1\}$, $\Im\{\mathbf{c}_1\}$, $\Re\{\mathbf{c}_2\}$ and $\Im\{\mathbf{c}_2\}$ gives

$$\begin{aligned}\frac{\partial \boldsymbol{\mu}^H}{\partial \Re\{\mathbf{c}_1\}} &= j \cdot \frac{\partial \boldsymbol{\mu}^H}{\partial \Im\{\mathbf{c}_1\}} = \\ &\quad [\boldsymbol{\Gamma}_N(\varepsilon)\mathbf{F}^H\mathbf{S}\mathbf{F}_{L_a}\mathbb{H}(1:L_a, 1:L_1)\boldsymbol{\Gamma}_{L_1}(-\varepsilon)]^H, \quad (43)\\ \frac{\partial \boldsymbol{\mu}^H}{\partial \Re\{\mathbf{c}_2\}} &= j \cdot \frac{\partial \boldsymbol{\mu}^H}{\partial \Im\{\mathbf{c}_2\}} = \\ &\quad [\boldsymbol{\Gamma}_N(-\varepsilon)\mathbf{F}\mathbf{S}^*\mathbf{F}_{L_b}^*\mathbb{H}(1:L_b, 1:L_2)\boldsymbol{\Gamma}_{L_2}(\varepsilon)]^H. \quad (44)\end{aligned}$$

Furthermore, differentiating (35) w.r.t. various parameters gives

$$\frac{\partial \mathbf{R_w}}{\partial \Re\{c_1(i)\}} = \mathbf{T}_i \mathbf{R_z} \mathbb{C}_1^H + \mathbb{C}_1 \mathbf{T}_i^T \mathbf{R_z}, \tag{45}$$

$$\frac{\partial \mathbf{R_w}}{\partial \Im\{c_1(i)\}} = j \cdot [\mathbf{T}_i \mathbf{R_z} \mathbb{C}_1^H - \mathbb{C}_1 \mathbf{T}_i^T \mathbf{R_z}], \tag{46}$$

$$\frac{\partial \mathbf{R_w}}{\partial \Re\{c_2(i)\}} = \mathbf{T}_i \mathbf{R_z} \mathbb{C}_2^H + \mathbb{C}_2 \mathbf{T}_i^T \mathbf{R_z}, \tag{47}$$

$$\frac{\partial \mathbf{R_w}}{\partial \Im\{c_2(i)\}} = j \cdot [\mathbf{T}_i \mathbf{R_z} \mathbb{C}_2^H - \mathbb{C}_2 \mathbf{T}_i^T \mathbf{R_z}], \tag{48}$$

$$\frac{\partial \mathbf{R_w}}{\partial \varepsilon} = \frac{\partial \mathbf{R_w}}{\partial \Re\{h(i)\}} = \frac{\partial \mathbf{R_w}}{\partial \Im\{h(i)\}} = \mathbf{0}_{N\times N}. \tag{49}$$

Substituting (37)-(39), (43)-(44) and (45)-(49) back into (34) and inverting the resultant $\mathbf{J}$, we can obtain the CRB for $\boldsymbol{\eta}$. Therefore, we have $\mathrm{CRB}(\varepsilon) = [\mathbf{J}^{-1}]_{11}$.

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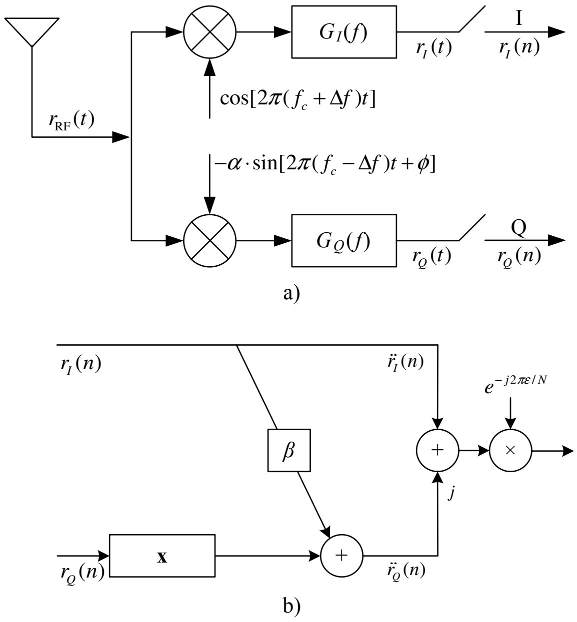

Fig. 1. a) Direct-conversion receiver (DCR) model; b) I/Q imbalances and CFO compensation structure (from [9])

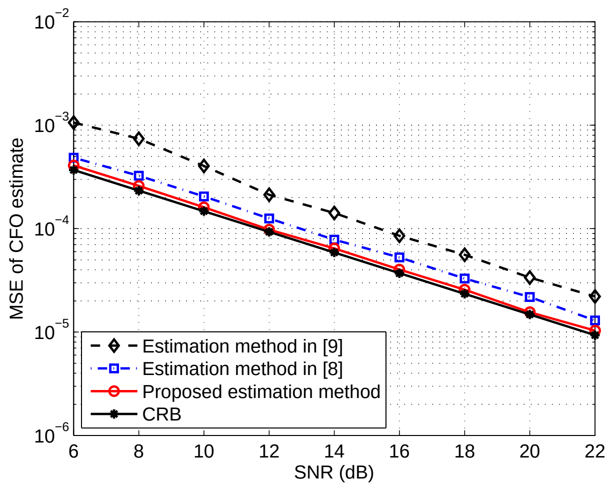

Fig. 2. CFO MSE versus SNR.

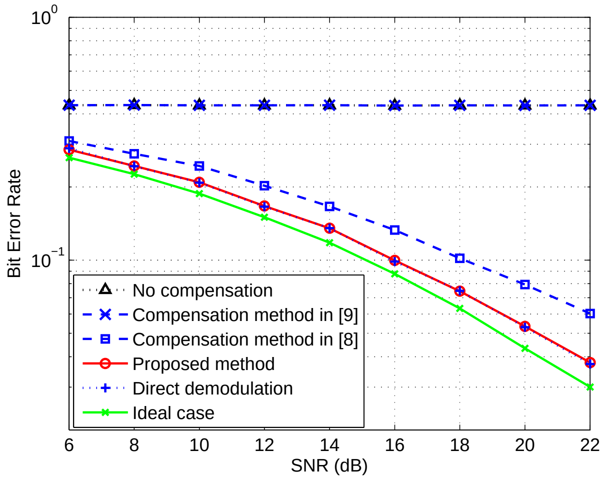

Fig. 3. BER versus SNR for different compensation schemes.

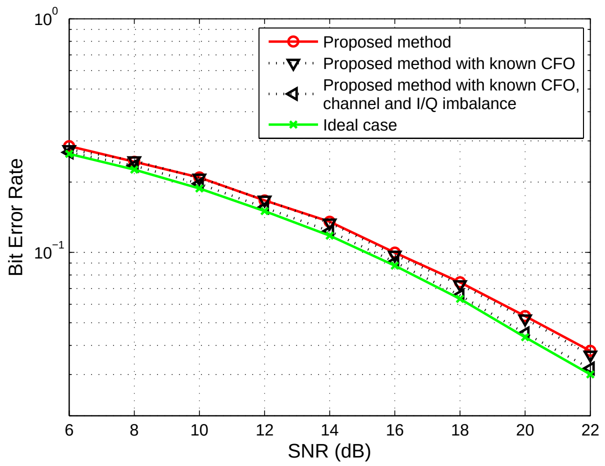

Fig. 4. BER versus SNR under different amount of CFO and channel state information.