

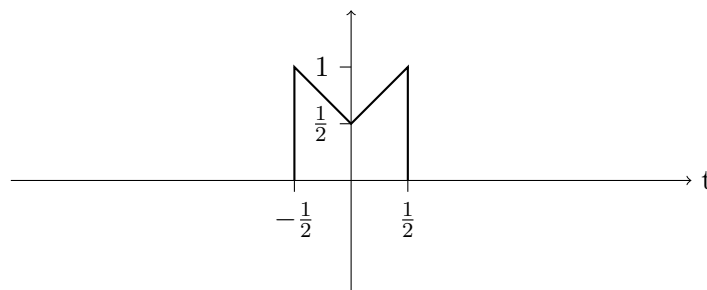
Name: _____

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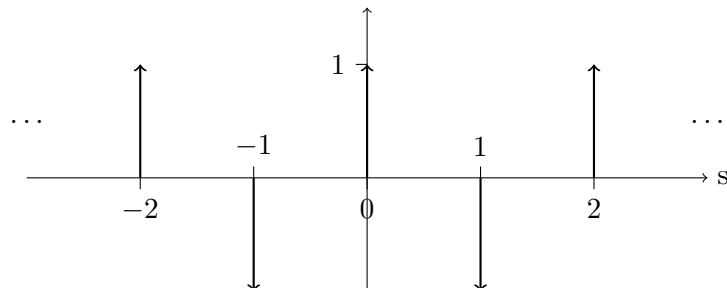
Test #1

Note: The test is closed books, closed notes. There are altogether 6 questions, with a total of 90 points. You have ninety minutes to complete it (so roughly one minute per point). Please write your answers *legibly*. Show your steps.

1. (10 points) Find the Fourier transform of the function $g(t)$ shown in the figure below.



2. (15 points) Evaluate and draw the Fourier transform of the “alternating impulses” shown below.



3. (20 points) Evaluate the Fourier transform of the two-dimensional function

$$f(x, y) = e^{-(a|x|+b|y|)},$$

where $a > 0$ and $b > 0$.

4. (15 points) With the help of the generalized Rayleigh's theorem, evaluate

$$\int_{-\infty}^{\infty} e^{-\pi t^2} \cos(2\pi at) dt.$$

5. (10 points) We have defined the Fourier transform to be

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi st} dt$$
$$f(t) = \int_{-\infty}^{\infty} F(s) e^{j2\pi st} ds$$

Now, suppose instead we define the forward Fourier transform to be

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{j\omega t} dt.$$

(Note that the sign of the exponent is positive.) What is the corresponding formula for the inverse Fourier transform?

6. (20 points) The short-time Fourier transform (STFT) of a signal $f(t)$ is defined as

$$\tilde{F}(\tau, s) = \int_{-\infty}^{\infty} f(t) w(t - \tau) e^{-j2\pi st} dt,$$

where $w(t)$ is a real-valued window with a unit area, i.e., $\int_{-\infty}^{\infty} w(t) dt = 1$.

- (a) Show how we can obtain $F(s)$, the Fourier transform of $f(t)$, from $\tilde{F}(\tau, s)$. *Hint: What do we get if we integrate $\tilde{F}(\tau, s)$ over τ ?*
- (b) Show how we can obtain $f(t)$ from $\tilde{F}(\tau, s)$.