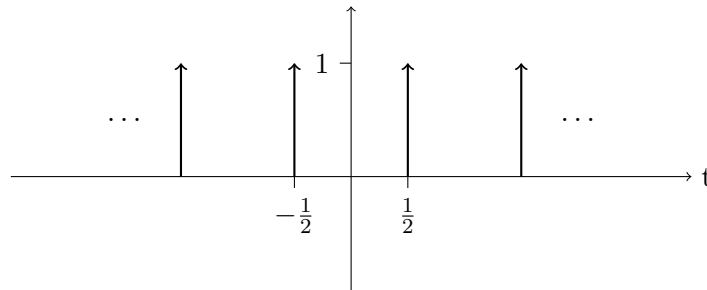


Test #1 Solution

1. The signal is $g(t) = \Pi(t) - \frac{1}{2}\Lambda(2t)$. The Fourier transform is therefore

$$G(s) = \text{sinc}(s) - \frac{1}{4}\text{sinc}^2\left(\frac{s}{2}\right).$$

2. One way to represent this function is $e^{j\pi t} \mathfrak{W}(t)$, which has a Fourier transform equal to $\mathfrak{W}(s - \frac{1}{2})$.



3. This is a separable function in the Cartesian coordinates, so let us first evaluate the 1D Fourier transform of $e^{-a|t|}$.

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-a|t|} e^{-j2\pi st} dt &= \int_{-\infty}^0 e^{at} e^{-j2\pi st} dt + \int_0^{\infty} e^{-at} e^{-j2\pi st} dt \\ &= \left[\frac{e^{(a-j2\pi s)t}}{a-j2\pi s} \right]_{-\infty}^0 + \left[\frac{e^{-(a+j2\pi s)t}}{-(a+j2\pi s)} \right]_0^{\infty} \\ &= \frac{1}{a-j2\pi s} + \frac{1}{a+j2\pi s} \\ &= \frac{2a}{a^2 + (2\pi s)^2} \end{aligned}$$

Therefore, the Fourier transform of $e^{-(a|x|+b|y|)}$ is

$$F(u, v) = \frac{4ab}{[a^2 + (2\pi u)^2][b^2 + (2\pi v)^2]}$$

4. With the help of the generalized Rayleigh's theorem,

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-\pi t^2} \cos(2\pi at) dt &= \int_{-\infty}^{\infty} e^{-\pi s^2} \cdot \frac{1}{2} [\delta(s+a) + \delta(s-a)] ds \\ &= \frac{1}{2} e^{-\pi a^2} + \frac{1}{2} e^{-\pi a^2} \\ &= e^{-\pi a^2} \end{aligned}$$

5. Let $f(t) = g(t)$. Then, we can see that

$$G(-2\pi s) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi st} dt = \int_{-\infty}^{\infty} f(t) e^{-j2\pi st} dt = F(s).$$

Now, using the inverse Fourier transform formula, we have

$$g(t) = f(t) = \int_{-\infty}^{\infty} F(s) e^{j2\pi st} ds = \int_{-\infty}^{\infty} G(-2\pi s) e^{j2\pi st} ds.$$

We perform a change of variable with $\omega = -2\pi s$, and after some manipulations, we have

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{-j\omega t} d\omega$$

6. (a) If we integrate $\tilde{F}(\tau, s)$ over τ , we have

$$\begin{aligned} \int_{-\infty}^{\infty} \tilde{F}(\tau, s) d\tau &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) w(t - \tau) e^{-j2\pi st} dt d\tau \\ &= \int_{-\infty}^{\infty} f(t) \left(\int_{-\infty}^{\infty} w(t - \tau) d\tau \right) e^{-j2\pi st} dt \\ &= \int_{-\infty}^{\infty} f(t) (1) e^{-j2\pi st} dt \\ &= F(s) \end{aligned}$$

$$\text{Therefore, } F(s) = \int_{-\infty}^{\infty} \tilde{F}(\tau, s) d\tau.$$

(b) Using the inverse Fourier transform formula,

$$\begin{aligned} f(t) &= \int_{-\infty}^{\infty} F(s) e^{j2\pi st} ds \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{F}(\tau, s) e^{j2\pi st} ds d\tau \end{aligned}$$