

Homework 1

{ Due: Sep 30, 2023, 11:55pm }

Instruction: Submit your answers electronically through Moodle. In Moodle, you must submit under the PDF Submission link with ONE PDF file containing answers to all remaining questions. Please ensure that you use notation symbols in alignment with the lecture notes.

Question 1

Find the Fourier transform of the following signal. (Hint: make good use of the $\wedge(\cdot)$ function)

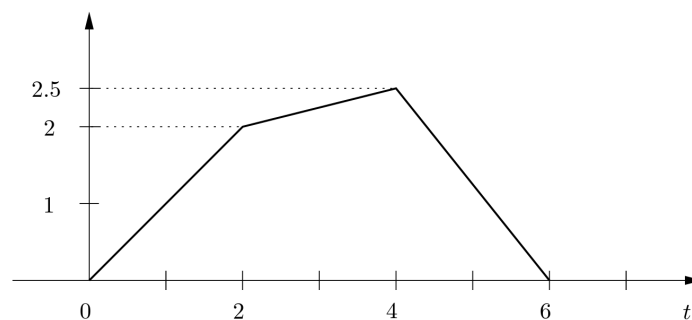


Figure 1: Time-domain representation of the signal.

Question 2

Consider a filter with real and even transfer function $H(s)$. If the input to the filter is $\cos(2\pi at)$, show that the output is $H(a)\cos(2\pi at)$.

Question 3

Let $f(t)$ be a signal with Fourier transform $F(s)$. Suppose we are given the following facts:

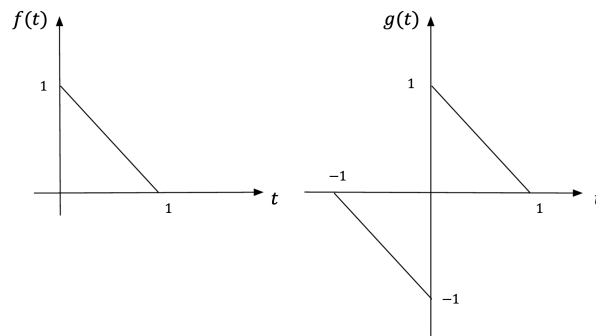
1. $f(t)$ is real
2. $f(t) = 0$ for $t \leq 0$
3. $\mathcal{F}^{-1}\{\text{Re}\{F(s)\}\} = |t|e^{-|t|}$, where $\text{Re}\{F(s)\}$ denotes the real part of $F(s)$.

Find $f(t)$. Hint: Recall that $\text{Re}\{F(s)\} = (F(s) + F^*(s))/2$

Question 4

Consider two function $f(t)$ and $g(t)$, shown in Figure 2. We denote their Fourier transforms as $F(s)$ and $G(s)$, respectively.

1. Without performing any integration, what is the real part of $F(s)$? Explain your reasoning. Hint: Consider separating the given function into its even and odd components.
2. It is known that the imaginary part of $F(s)$ is $(\sin(2\pi s) - 2\pi s)/(4\pi^2 s^2)$. Without performing any integration, find the expression of $G(s)$.

Figure 2: Graph of $f(t)$ and $g(t)$

Question 5

Determine the Fourier transforms of the following functions, and answer questions:

1.

$$f_1(t) = \begin{cases} \frac{1}{N} \left(1 - \frac{|t|}{N}\right) & \text{when } |t| \leq N \\ 0 & \text{when } |t| > N \end{cases}$$

2.

$$f_2(t) = \frac{1}{N} e^{-\frac{t}{N}} H(t)$$

3.

$$f_3(t) = \frac{1}{N} e^{\frac{t}{N}} H(-t)$$

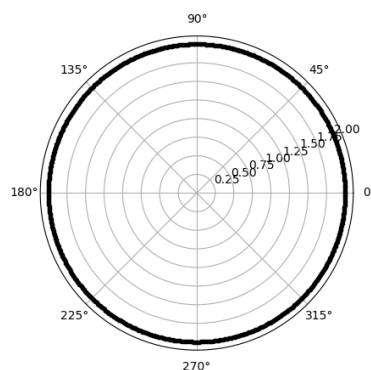
4.

$$f_4(t) = \frac{1}{2N} e^{-\frac{|t|}{N}}$$

What limits do the Fourier transforms of $f_1(t)$ and $f_4(t)$ approach when N approaches 0? Explain these results and make a conclusion of what you find.

Question 6

Consider an ideal ring (the thickness is infinitesimal) in a polar coordinate system, shown in Figure 3.

Figure 3: A specific instance of an ideal ring with its radius $r_0 = 2$.

1. What is the mathematical expression of an ideal ring with a radius of r_0 in the polar coordinate system?
2. Find its Fourier-Bessel transform with a detailed derivation. Please follow the notations in the course notes.

Question 7

Consider an infinite 2D grating mask with the spatial period denoted as (X, Y) , whose repeated unit is a 2D rect function with its side length of (a, a) and a unit strength, shown in Figure 4.

1. What is the mathematical expression of the transmittance of this mask, i.e., the white gates in Figure 4, in the 2D rectangular coordinate system?
2. Find the 2D Fourier transform of the transmittance of this mask with a detailed derivation. Please follow the notations in the course note.

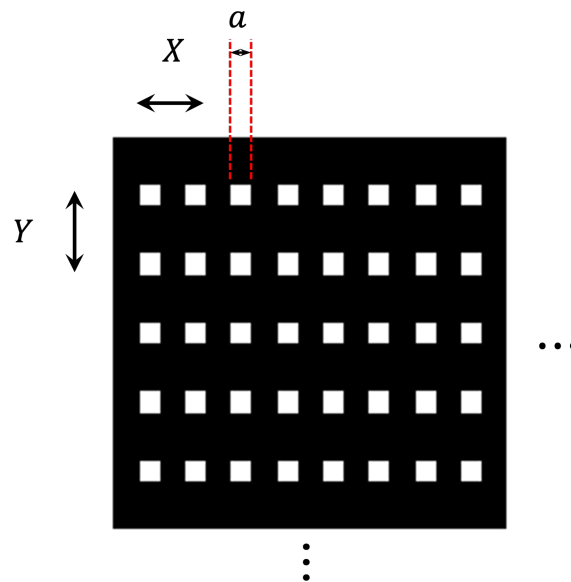


Figure 4: An infinite 2D grating mask.